

Problem 1. (16 pts) Solve:

$$y'' + y' - 6 = 0, \quad y(0) = 2, \quad y'(0) = 9.$$

Problem 2. (14 pts) Using the method of undetermined coefficients, find the general solution to the differential equation

$$2 \cdot y'' - y' = t$$

Problem 3. (26 pts total) **Throughout this problem**, which has parts (a)-(c), consider the system $x' = Ax$ where the matrix $A = \begin{pmatrix} -2 & 2 \\ -10 & -10 \end{pmatrix}$ has an eigenvalue $-6 + 2i$ with eigenvector $\begin{pmatrix} 1 \\ i - 2 \end{pmatrix}$.

(a) (8 pts) Find a **fundamental set of REAL** solutions of the system $x' = Ax$.

(b) (8 pts) Find the solution of the system described on top of page 3 satisfying $x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(b) (10 pts) Find and classify the type and stability of the equilibrium points for the system described on top of page 3. Sketch the phase portrait and make sure you sketch a couple of solutions with arrows on them to illustrate the direction of the flow and explain their behavior as $t \rightarrow \infty$ and $t \rightarrow -\infty$.

Problem 4. (16 pts) Solve:

$$\begin{cases} x_1' = -x_2 \\ x_2' = x_1 + 2x_2 \end{cases}$$

Problem 5. (14 pts) Using variation of parameters, solve:

$$t^2 y'' - 2ty' + 2y = t^2, \quad \text{where } t < 0.$$

You may use, without a proof, the fact that $y_1(t) = t$ and $y_2(t) = t^2$ are solutions of the homogenous equation.

Problem 6. (14 pts) The undamped system

$$\frac{3}{2}x'' + kx = 0, \quad x(0) = 0, \quad x'(0) = v_0$$

is observed to have amplitude 3 and frequency $5/\pi$. Find k and the initial velocity v_0 .