Syllabus: Stanford PhD Qualifying Exam in Algebra

I. Group Theory. Subgroups, Normal Subgroups, Quotient Groups, Isomorphism Theorems, Simple Groups, Jordan Hölder Theorems, Sylow Theorems and applications, group actions, symmetric and permutation groups, linear groups ($GL(n, F)$ and $SL(n, F)$), group extensions, semidirect product, solvable and nilpotent groups, p-groups, finitely generated abelian groups, free groups and group presentations.

II. Linear Algebra. Dimension, linear transformations, eigenvalues and eigenvectors, characteristic and minimal polynomial, Cayley-Hamilton theorem, trace and determinants, diagonalization, rational canonical form, Jordan canonical form, bilinear forms, multilinear algebra, tensor products, symmetric and exterior powers, Hermitian forms, spectral theorem for finite dimensional inner product spaces, orthogonal and unitary groups.

III. Rings and Modules. Euclidean domains, principal ideal domains, unique factorization domains, polynomial rings, power series rings, ideals, ring homomorphisms, prime and maximal ideals, Chinese remainder theorem, chain conditions, Noetherian rings and modules, finitely generated modules over a principal ideal domain, projective and injective modules, local rings, localization, Nakayama's lemma, discrete valuation rings, integral ring extensions, Dedekind domains, Hilbert basis theorem, Nullstellensatz, semisimple rings and modules, matrix rings, division rings.

IV. Field Theory. Finite extensions, degree of an extension, algebraic and transcendental extensions, normal and separable extensions, theorem of the primitive element, splitting fields, algebraic closure, field embeddings and automorphisms, Galois theory, inseparable extensions, number fields, finite fields, solvability by radicals, Hilbert theorem 90, norms and traces, transcendence degree, function fields.

V. Group Representation Theory. Irreducible representations, Maschke's theorem, semisimple group rings, Schur's lemma, characters, Schur orthogonality, character tables, complex, real and rational representations, induced representations, Frobenius reciprocity, Burnside's $p^a q^b$ theorem, Frobenius' theorem on Frobenius groups.

References

Lang's *Algebra*, Jacobson's *Basic Algebra I and II*, Rotman's *Advanced Modern Algebra*.