1. We can think of the elements of $G$ as points of the $xy$-plane: $G$ is then all of the plane except for the $x$-axis. The left coset of $H$ containing $[x, y]$ is $[x, y]H$, i.e.,

$$\{[x, y][h, 0] : h \neq 0\} = \{[xh, y] : h \neq 0\}$$

which is the horizontal line through $[x, y]$. The right coset of $H$ containing $[x, y]$ is

$$H[x, y] = \{[h, 0][x, y] : h \neq 0\} = \{[hx, hy] : h \neq 0\},$$

which is the line through $[x, y]$ and the origin (excluding the origin, which is not in $G$.) The left coset of $N$ containing $[x, y]$ is

$$[x, y]N = \{[x, y][1, t] : t \in \mathbb{R}\} = \{[x, xt + y] : t \in \mathbb{R}\},$$

which is the vertical line through $[x, y]$. The right coset of $N$ containing $[x, y]$ is $N[x, y]$, i.e.,

$$\{[1, t][x, y] : t \in \mathbb{R}\} = \{[x, y + t] : h \neq 0\}$$

which is also the vertical line through $[x, y]$.

Thus $N$ is a normal subgroup of $G$ but $H$ is not.

2(i). 2 cards : $(1 \ 2)$
4 cards : $1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \equiv 3 \pmod{5} \rightarrow 6 \equiv 1$, so

$$\sigma = (1 \ 2 \ 4 \ 3)$$

6 cards : $1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \equiv 1 \pmod{7}$,

$$3 \rightarrow 6 \rightarrow 12 \equiv 5 \rightarrow 10 \equiv 3$$, so

$$\sigma = (1 \ 2 \ 4)(3 \ 6 \ 5)$$
8 cards : $1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \equiv 7 \pmod{9} \rightarrow 14 \equiv 5 \rightarrow 10 \equiv 1$,

$$3 \rightarrow 6 \rightarrow 12 \equiv 3$$, so

$$\sigma = (1 \ 2 \ 4 \ 8 \ 7 \ 5)(3 \ 6).$$

(ii) If a deck of $2n$ cards is perfectly shuffled $k$ times, card $j$ goes to position $2^k j \pmod{2n + 1}$. So the deck will be back to its original order if and only if $2^k \equiv 1 \pmod{2n + 1}$, i.e., if and only if

$$2^k = (2n + 1)j + 1 \quad \text{(for some } j)$$

i.e., if and only if

$$(*) \quad 2n = \frac{2^k - 1}{j} - 1.$$
for some integer $j$. We want $n$ to be as large as possible, so we make $j$ as small as possible, namely $j = 1$. This gives $2n = 2^k - 2$.

In other words: a deck of $2^k - 2$ cards will return to its original order after $k$ perfect shuffles (but no larger deck will).

Consider, for example $k = 6$. The equation (*) becomes:

$$2n = \frac{63}{j} - 1.$$ 

Thus the largest deck size is 62. The next largest (corresponding to $j = 3$) is 20, then 8 ($j = 7$), then 6 ($j = 9$.)