Problem 1
Let \( B = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix} \)

(a) Compute the matrix \( B^2 \).
(b) Find the inverse (if it exists) of the matrix
\[ I_4 - B = \begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1
\end{bmatrix} \]

Problem 2
Let
\[ A = \begin{bmatrix}
1 & 1 & 2 \\
3 & 2 & 3 \\
2 & 1 & 2
\end{bmatrix} \]

Is \( A \) invertible? If so, find \( A^{-1} \), showing all reasoning. If not, explain why not.

Problem 3
For the following question, please show your work.
(a) Let \( \beta = \{v_1, v_2\} \), where \( v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \) and \( v_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \). Transform the following vectors (shown in standard coordinates) into coordinates with respect to the basis \( \beta : \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \).
(b) Let \( A \) be such a \( 2 \times 2 \) matrix that \( Av_1 = -v_1 \) and \( Av_2 = 2v_2 \). Find \( A \begin{bmatrix} 2 \\ 1 \end{bmatrix}, A \begin{bmatrix} 0 \\ 1 \end{bmatrix}, A \begin{bmatrix} 1 \\ 0 \end{bmatrix} \).
(c) Write down the matrix for \( A \) (in the standard basis).

Problem 4
(a) Let \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) be a linear transformation given by \( T(x) = Ax \), where \( A \) is a \( 2 \times 2 \) matrix; and suppose we know that \( T \left( \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) and \( T \left( \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \)

Find \( A \); show all your reasoning.
(b) Find, with justification, a \( 2 \times 2 \) matrix \( M \) such that \( M \neq I_2, M^2 \neq I_2 \), and \( M^3 \neq I_2 \), but \( M^4 = I_2 \). (Here \( I_2 \) is the \( 2 \times 2 \) identity matrix.)

Problem 5
Let \( \{u, v\} \) be the linear coordinates on \( \mathbb{R}^2 \) with respect to the basis \( B = \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} \).

Express \( u \) and \( v \) in terms of \( x \) and \( y \), and also express \( x \) and \( y \) in terms of \( u \) and \( v \). Use the latter to express the equation \( x^2 + y^2 = 1 \) in terms of \( \{u, v\} \)-coordinates; your answer should be \( au^2 + buv + cv^2 = 1 \) for some integers \( a, b, c \).

Midterm review problems: Problem #3(a), #8, Midterm 2, Winter 2015 (skip the first two functions), Problem #8, #11(a) Final Exam, Spring 2014, Problem #5, Midterm 2, Winter 2012.