You can find solutions to all section worksheets at: http://math.stanford.edu/~valentin/Math51/

Problem 1
True or False?
1. If $A$ is a $4 \times 2$ matrix then $\dim N(A) \leq 2$.
2. If $A$ is a $2 \times 4$ matrix then $\dim N(A) \geq 2$.
3. There are $3 \times 6$ matrices with $\dim N(A) = 3$ and $\dim C(A) = 3$.
4. There are $6 \times 3$ matrices with $\dim N(A) = 3$ and $\dim C(A) = 3$.

Problem 2
Let $T_1: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation which projects onto the line spanned by the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
1. Find the matrix, $A_1$, which represents $T_1$.
2. What is $C(A_1)$?

Problem 3
Define $T_2: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ to be reflection over the $x$-axis. Find the matrix which represents $T_2$.

Problem 4
Define $T_3: \mathbb{R}^2 \rightarrow \mathbb{R}^4$ to be a linear transformation such that
$$
T \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \end{bmatrix}, \quad T_3 \left( \begin{bmatrix} 0 \\ 1 \\ \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}
$$

Midterm Review.
True or False?
1. Suppose that $A$ is an $n \times n$ matrix and that its null space consists of a single point. Then every inhomogeneous equation $Ax = b$ has a unique solution.
2. If $A$ is a matrix and the equation $Ax = b$ has at least two solutions, then the set of solutions contains a plane.
3. If $A$ is a $5 \times 8$ matrix, there is a vector $b \in \mathbb{R}^5$ so that the equation $Ax = b$ has a unique solution.
4. The set $\{(n, m) \mid n$ and $m$ are integers$\}$ is a subspace of $\mathbb{R}^2$.
5. The set $\{(x, y) \in \mathbb{R}^2 \mid xy = 0\}$ is a subspace of $\mathbb{R}^2$.
6. If the set $\{v_1, \ldots, v_k\}$ spans $V$, then the set $\{T(v_1), \ldots, T(v_k)\}$ will span $T(V)$, where $T$ is a linear transformation.
7. If the set $\{v_1, \ldots, v_k\}$ is a basis of $V$, then the set $\{T(v_1), \ldots, T(v_k)\}$ is a basis of $T(V)$, where $T$ is a linear transformation.
8. The span of four vectors in $\mathbb{R}^5$ forms a four dimensional subspace.

Find a basis for the column space and null space of the following matrix: $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$.

What is the rank and nullity of $A$? What does this say about existence and uniqueness of solutions to the equation $Ax = b$?