Problem 1
Suppose \( V \) is a subset of \( \mathbb{R}^n \).

1. List the three properties that \( V \) must have in order to be a linear subspace of \( \mathbb{R}^n \).

2. Which of the following are linear subspaces of \( \mathbb{R}^2 \)? Please explain your answer.
   - The set \( V = \{ (x, y) \in \mathbb{R}^2 \mid x + y \leq 0 \} \).
   - The set \( W = \{ (x, y) \in \mathbb{R}^2 \mid xy \geq 0 \} \).

Problem 2
Consider the following set:

\[
V = \{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 - x_2 + x_3 = 0 \}.
\]

1. Show that \( V \) is a linear subspace.

2. Find a basis for \( V \). What is the dimension of \( V \)?

3. Give an example of a matrix \( A \) such that \( \text{N}(A) = V \).

4. Give an example of a matrix \( A \) such that \( \text{C}(A) = V \).

Problem 3
Let \( A = \begin{bmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \).

1. Find all solutions to the equation \( Ax = \begin{bmatrix} 3 \\ 5 \\ -7 \end{bmatrix} \).

2. Find a basis for \( \text{N}(A) \).

3. Find a basis for \( \text{N}(A) \) that contains the vector \( \begin{bmatrix} 11 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix} \), or state why no such basis exists.

4. Find a basis for \( \text{C}(A) \) that contains the vector \( \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \), or state why no such basis exists.

5. What is \( \dim(\text{C}(A)) \)? How about \( \dim(\text{N}(A)) \)? What is their sum? How would you compute their sum if you didn’t know each of them individually?

Problem 4
Consider the pair of equations

\[
\begin{align*}
x + 4y + 5az &= -2 \\
3x + 5y + az &= 1
\end{align*}
\]

in \((x, y, z)\) with the coefficients of \( z \) involving the unspecified number \( a \).
1. Assume $a = 2$. In this case, give a parametric formula for the solutions of this pair of equations. Your answer should be written in the form of a parameterization of a line.

2. Compute an analogous parametric formula for every value of $a$ (i.e., parametrize the solutions in a manner that works for every value of $a$); this should recover your answer to the previous part upon setting $a = 2$. Again, your answer should be written in the form of a parameterization of a line.

**Problem 5** The matrix $A$ below has the given reduced row echelon form (You don’t need to verify this):

$$A = \begin{bmatrix}
3 & 6 & 1 & 17 & 3 \\
2 & 4 & 1 & 12 & 3 \\
4 & 8 & -1 & 18 & -3 \\
7 & 14 & -10 & 15 & -30
\end{bmatrix}, \quad \text{rref}(A) = \begin{bmatrix}
1 & 2 & 0 & 5 & 0 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

1. Find a basis for the column space $C(A)$ and the null space $N(A)$ of $A$.

2. Given that $A \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 8 \\ 10 \\ 1 \end{bmatrix}$ find all solutions of $Ax = \begin{bmatrix} 11 \\ 8 \\ 10 \\ 1 \end{bmatrix}$.

**Midterm preparation.** Review the concepts and definitions and answer the questions from Problem 8 from Midterm 1, Autumn 2014.