Problem 1
1. Complete the following sentence: A set of vectors \( \{v_1, \ldots, v_k\} \) is defined to be linearly dependent if..

2. Suppose that \( x \) belongs to \( \text{Span}(u, v, w) \), the subspace of \( \mathbb{R}^4 \) spanned by nonzero vectors \( u, v, w \); and suppose further that \( x \) is orthogonal to each of the vectors \( u, v, \) and \( w \). Show that \( x = 0 \).

3. Find a linear dependence of the three vectors below, or prove that they are independent.
\[
\begin{bmatrix}
1 \\
2 \\
4
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
2 \\
1
\end{bmatrix}, \quad
\begin{bmatrix}
-2 \\
5 \\
7
\end{bmatrix}.
\]

Problem 2
1. Suppose that \( ||a|| = 3, ||b|| = 2 \) and \( a \cdot b = 2 \). Calculate \( (3a + 5b) \cdot (a - 2b) \) and find the angle between \( a \) and \( b \). (You can leave your answer in the form of an inverse sine, cosine or tangent function, for example.)

2. What does it mean for two vectors \( a \) and \( b \) in \( \mathbb{R}^n \) to be orthogonal to one another? State your answer as a mathematical formula, not in words.

3. Suppose \( a, b \) are two nonzero vectors in \( \mathbb{R}^n \). Show that they have the same magnitude (i.e., \( ||a|| = ||b|| \)) if and only if \( a - b \) and \( a + b \) are orthogonal.

Problem 3 Let \( P \) be the parallelogram with vertices \((0, 0, 0), (1, 0, 1), (2, 2, 1), (1, 2, 0)\). Find its area using the cross product.

Problem 4 Let \( P \) be the plane in \( \mathbb{R}^3 \) spanned by the vectors
\[
\begin{bmatrix}
1 \\
-1 \\
1
\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}
1 \\
0 \\
2
\end{bmatrix}.
\]

1. Find a nonzero vector which is normal to \( P \).

2. Write down the equation for the plane \( P \) in the form \( ax + by + cz = d \).

3. Let \( Q \) be a plane parallel to \( P \) that contains the point \((1, 0, 0)\). Write down a parametric representation of the plane \( Q \).

4. Let \( L \) be the line spanned by \( \begin{bmatrix}
1 \\
-1 \\
1
\end{bmatrix} \). Find a parametric equation of a plane \( S \) whose intersection with \( P \) is \( L \).

Problem 5 (LA 6.5) : Solve the system of linear equations by expressing the given system as an augmented matrix and finding its rref:

\[
\begin{align*}
u + 2v + 3w &= -1 \\
u + 2v + 4w &= -2 \\
-2u - 4v - 4w &= 2
\end{align*}
\]