Problem 1
What geometric objects are the following sets:

1. Span \( \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \)

2. Span \( \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\} \)

3. Span \( \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \)

Problem 2
1. Is it true that
   \[
   \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \right\} = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}?
   \]
   Prove why or why not.

2. Draw \( \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \right\} \). What is the shape? Does it look like \( \mathbb{R}, \mathbb{R}^2, \mathbb{R}^3 \)? Make a connection between this and the number of vectors in the smallest ‘spanning set.’

Problem 3
1. Complete the following sentence: A set of vectors \( \{v_1, \ldots, v_k\} \) is defined to be linearly independent if.

2. Interpret/explain the answer to the previous problem in terms of linear independence.

3. Is the following statement true or false: In a set of linearly dependent vectors we can write any vector as a linear combination of the others.

Problem 4
1. Show that the following set of vectors is linearly independent.
   \[
   \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}
   \]

2. Suppose the set of vectors \( \{u, v, w\} \) in \( \mathbb{R}^7 \) is linearly independent. Is \( \{3u+2v, v, u+v+w\} \) necessarily linearly independent? How about the set \( \{2u + v - w, v - w, u - v + w\} \)?

Challenge problem
1. Let \( u, v, w \) be linearly independent vectors in \( \mathbb{R}^{12} \). Show that if there are vectors \( a, b, c \) such that
   \[
   \text{Span} \{u, v, w\} = \text{Span} \{a, b, c\}
   \]
   then the vectors \( a, b, c \) are linearly independent.

2. Show that the solutions to the equation \( x + y + z = 1 \) form a plane.