Fundamental Theorem of Calculus: If $f$ is continuous on $[a,b]$ and $F$ is an antiderivative of $f$, then
\[ \int_a^b f(x)dx = F(b) - F(a) \]

The substitution rule: If $g'$ is continuous on $[a,b]$ and $f$ is continuous on the range of $g(x) = u$, then
\[ \int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du \]

Problem 1. Compute the following indefinite integrals:
1. \[ \int e^t \sqrt{1 + e^t} dt \]
2. \[ \int x(2x + 3)^3 dx \]
3. \[ \int \frac{\sin 2x}{1 + \cos^2 x} dx \]
4. \[ \int \frac{1 + x}{1 + x^2} dx \]
5. \[ \int \sqrt[3]{4 + 5x} dx \]

Problem 2. Compute the following definite integrals:
1. \[ \int_0^1 \sin \left( \frac{\pi t}{2} \right) dt \]
2. \[ \int_{-2}^2 (2x + 3)^3 dx \]
3. \[ \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx \]
4. \[ \int_e^{e^2} \frac{dx}{x \sqrt{\ln x}} \]

Problem 3. Write the following limit as an integral (write it in two ways if you can and explain why the results are equal. do not compute the integrals.):
\[ \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{2n} \cos \left( \frac{i}{2n} \right) \sin \left( \frac{i}{n} \right) \]
**Bonus Problem**

a) Compute the following definite integral:

\[ \int_{-1}^{1} \left( \frac{1}{\sqrt{x}} \right)^4 \, dx \]

b) Draw a very rough graph of the function above. What do you notice?