Stanford Algebraic Geometry Seminar

POLYGON SPACES AND MIRROR SYMMETRY

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Abstract:

The possible configurations, up to orientation-preserving isometry, for a planar n-gon with prescribed length for each of its edges, make-up a compact space, which is, in general, a smooth, orientable manifold of dimension $(n-3)$. Its topological type varies according a chamber structure for admissible edge-length-vectors, and can be investigated by means of Morse theory, geometric invariant theory, symplectic and toric geometry.
In adequate coordinates, the defining equations are algebraic, and yield families of complex projective varieties whose real points are the above configuration spaces. In particular, a construction used by Darboux for quadrilaterals, leads, in arbitrary dimension, to Calabi-Yau varieties. The singularities of the latter are away from the real locus, and resolutions to Calabi-Yau manifolds will contain identifiable types of special Lagrangians.
These resolved Darboux varieties can be presented as codimension two complete intersections in the toric variety associated to a permutohedron.
A conjecture of Strominger, Yau, and Zaslow suggests that mirror symmetry for pairs of Calabi-Yau manifolds corresponds geometrically to a duality of fibrations in special Lagrangian tori, and indeed, we do find special Lagrangian tori at appropriate points in our families.
We also detect the mirror families (using Batyrev-Borisov duality) and retrieve the polygon spaces as special Lagrangians on the mirror Calabi-Yau manifolds.
These phenomena are related to the root system A_{n-1}, and there are similar developments for the BC_n type. This is enough for propagating our examples to a significantly larger class of Calabi-Yau manifolds which arise in connection with certain reflexive centrally symmetric polytopes.
A different, yet related complexification, and thus other examples of special Lagrangian tori on Calabi-Yau manifolds, can be obtained from the non-Euclidean scenario. Up to birational transformations, the Euclidean case appears as a limit of the non-Euclidean case.

Friday, January 10, 3:30 pm
Room 383–N

`http://math.stanford.edu/~vakil/seminar.html`