Abstract:
A pencil of genus $b$ of curves of genus $g$ stands for a morphism $f : S \to B$ onto a smooth curve of genus $b$, and with connected fibres of arithmetic genus $g$. From the birational point of view, we have a curve of genus $g$ over the function field of $B$. Therefore, we understand birationally the case where $g \leq 5$ pretty much. From the biregular point of view, the naive hope would be to embed $S$ into a projective bundle over $B$. To make things work and be precise, we consider the relative canonical algebra $R$ of $f$. After recalling the Arakelov-Fujita inequalities for the vector bundles which are the graded pieces of $f$, we discuss recent results (1-2-3- theorem) on the generation of $R$ (results of Konno, Franciosi). Finally, we shall discuss structure theorems for $R$ in the case where $1 < g < 5$. Applications are given, e.g. to the classification of Godeaux surfaces (joint work with Roberto Pignatelli).