1. A game starts with four heaps of beans, containing 3, 4, 5 and 6 beans. The two players move alternately. A move consists of taking either

(a) one bean from a heap, provided at least two beans are left behind in that heap, or
(b) a complete heap of two or three beans.

The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy. (Nathan Pflueger, 1995B5)

2. Consider integers 1, 2, 3, ..., 2n and pick more than n of them. Show that regardless of the choice made, one can find two integers picked such that one divides another. (Kiyoto Tamura, a problem from Erdos)

3. Let \( S = \{1, 2, \ldots, n\} \) and let \( S_1, S_2, \ldots, S_n \) be subsets of \( S \) satisfying \( |S_i \cap S_j| \leq 1 \) for \( i \neq j \). Show that \( \max(|S_1| + |S_2| + \cdots + |S_n|) \) is asymptotically \( n \sqrt{n} \). (Bob Hough)

4. Evaluate

\[
\int_0^2 \left( (x^3 + 1)^{1/2} + (x^2 + 2x)^{1/3} \right) \, dx.
\]

(Natth Bejraburnin)

5. Evaluate

\[
\sqrt[8]{\frac{2207 - 1}{2207 - \frac{1}{2207}}}.
\]

Express your answer in the form \( \frac{a + b \sqrt{c}}{d} \), where \( a, b, c, d \) are integers. (Ravi Vakil, 1995B4)