1. Let \( p(x) \) be a nonzero polynomial of degree less than 1992 having no nonconstant factor in common with \( x^3 - x \). Let

\[
\frac{d^{1992}}{dx^{1992}} \left( \frac{p(x)}{x^3 - x} \right) = \frac{f(x)}{g(x)}
\]

for polynomials \( f(x) \) and \( g(x) \). Find the smallest possible degree of \( f(x) \). (Kiat Chuan Tan, 1992B4)

2. Let \( A(n) \) denote the number of sums of positive integers \( a_1 + a_2 + \cdots + a_r \) which add up to \( n \) with \( a_1 > a_2 + a_3, a_2 > a_3 + a_4, \ldots, a_{r-2} > a_{r-1} + a_r, a_{r-1} > a_r \). Let \( B(n) \) denote the number of \( b_1 + b_2 + \cdots + b_s \) which add up to \( n \) with

(i) \( b_1 \geq b_2 \geq \cdots \geq b_s \),

(ii) each \( b_i \) is in the sequence \( 1, 2, 4, \ldots, g_j, \ldots \) defined by \( g_1 = 1, g_2 = 2, \) and \( g_j = g_{j-1} + g_{j-2} + 1 \), and

(iii) if \( b_1 = g_k \) then every element in \( \{1, 2, 4, \ldots, g_k\} \) appears at least once as a \( b_i \).

Prove that \( A(n) = B(n) \) for each \( n \geq 1 \).

For example, \( A(7) = 5 \) because the relevant sums are \( 7, 6 + 1, 5 + 2, 4 + 3, 4 + 2 + 1, 2 + 2 + 2 + 1, 2 + 2 + 1 + 1 + 1, 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \). (John Hegeman, Putnam 1991A6)

3. Show that for every positive integer \( n \),

\[
\left( \frac{2n-1}{e} \right)^{2n-1} < 1 \cdot 3 \cdot 5 \cdots (2n-1) < \left( \frac{2n+1}{e} \right)^{2n+1}.
\]

(Nathan Pflueger, Putnam 1996B2)

4. Suppose \( x_1, \ldots, x_n \) are each \( \pm 1 \), and

\[
\sum_{i=1}^{n} x_i x_{i+1} x_{i+2} x_{i+3} = 0,
\]

where \( x_{n+1} \) is interpreted as \( x_1 \). Show that \( n \) is divisible by 4. (Ravi Vakil)

5. Prove that if

\[
11z^{10} + 10iz^9 + 10iz - 11 = 0,
\]

then \( |z| = 1 \). (Here \( z \) is a complex number and \( i^2 = -1 \).) (Kiat Chuan Tan, 1989A3)

6. Show that there is some \( N \) so that if you take any \( N \) points on the plane, no three on a line, then you can find 2005 of them that form a convex 2005-gon. (Ravi Vakil, from a

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book by Polya, Tarjan, and Woods — Polya was professor here long ago, famous for his books on problem-solving

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