1. Consider two lists. List A consists of the positive powers of 10 (10, 100, 1000, …) written in base 2. List B consists of the positive powers of 10 written in base 5. Show that, for any integer \( n > 1 \), there is exactly one number in exactly one of the lists that is exactly \( n \) digits long.

<table>
<thead>
<tr>
<th>Powers of 10</th>
<th>List A</th>
<th>List B</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1010 (4 digits)</td>
<td>20 (2 digits)</td>
</tr>
<tr>
<td>100</td>
<td>1100100 (7 digits)</td>
<td>400 (3 digits)</td>
</tr>
<tr>
<td>1000</td>
<td>1111101000 (10 digits)</td>
<td>13000 (5 digits)</td>
</tr>
<tr>
<td>10000</td>
<td>1001110001000 (14 digits)</td>
<td>310000 (6 digits)</td>
</tr>
</tbody>
</table>

(Ravi Vakil, a problem I made up long ago, that appeared on the 1994 Asian Pacific Mathematical Olympiad)

2. Let \( f(x) \) be differentiable on \([0, 1]\) with \( f(0) = 0 \) and \( f(1) = 1 \). For each positive integer \( n \) and arbitrary given positive numbers \( k_1, k_2, \ldots, k_n \), show that there exist distinct \( x_1, x_2, \ldots, x_n \) such that

\[
\frac{k_1}{f'(x_1)} + \frac{k_2}{f'(x_2)} + \cdots + \frac{k_n}{f'(x_n)} = k_1 + k_2 + \cdots + k_n.
\]

(Bob Hough, from Larson 6.6.9)

3. For a positive real number \( r \), let \( G(r) \) be the minimum value of \( |r - \sqrt{m^2 + 2n^2}| \) for all integers \( m \) and \( n \). Prove or disprove the assertion that \( \lim_{r \to \infty} G(r) \) exists and equals 0.

(Theo Johnson-Freyd, Putnam 1986 B4)

4. The first \( 2n \) natural numbers are arbitrarily divided into two groups of \( n \) numbers each. The numbers in the first group are sorted in ascending order, i.e., \( a_1 < \cdots < a_n \), and the numbers in the second group are sorted in descending order: \( b_1 > \cdots > b_n \). Find, with proof, the sum

\[
|a_1 - b_1| + \cdots + |a_n - b_n|.
\]

(Paul-Olivier Dehaye)

5. Consider a regular \( n \)-gon inscribed in a unit circle with vertices labeled (cyclically) \( P_1, \ldots, P_n \). Show that

\[
|P_1P_2| |P_1P_3| \cdots |P_1P_n| = n.
\]

(Ravi Vakil)

Date: Monday, October 17, 2005.