PROBLEM-SOLVING MASTERCLASS WEEK 4

1. A positive integer is alternating if every two consecutive digits in its decimal representation are of different parity. Find all positive integers n such that n has a multiple which is alternating. (IMO2004 # 6, Alok Aggarwal)

2. Prove that, for any natural number n, there exists an arrangement of 1×1 squares in the plane that can be tiled with 1×2 dominoes in exactly n different ways. (Roger Grosse)

3. Sum the series

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n 3^m + m 3^n)}.$$

(1999A4, Alex Chen)

4. Given x, y, z real numbers with

$$\begin{aligned} x + y + z &= 3, \\ x^2 + y^2 + z^2 &= 25, \\ x^4 + y^4 + z^4 &= 209, \end{aligned}$$

Find $x^{100} + y^{100} + z^{100}$. (Bob Hough)

5. Suppose x and y are integer solutions of the equation

$$2x^2 + x = 3y^2 + y.$$

Prove that $x - y$ and $2x + 2y + 1$ are perfect squares. (Ivan Ivan Janatra)

E-mail address: vakil@math.stanford.edu

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