Responses to Third Referee Report for
"Equations for the moduli space of n points on the line"
by B. Howard, J. Millson, A. Snowden, R. Vakil

Please thank the referee for the suggestions, which clearly required a very careful and thorough reading of the preprint.

The referee suggested that we rephrase the Main Theorem so it states the stronger result that we actually prove. This is an excellent suggestion, and we have done so, making necessary changes in statements elsewhere in the paper.

Responses to points # 1 to # 25.

1. We added a reference (Kapovich and Millson) that the set of singular points is contained within the finite set of strictly semistable points.

2. This is now done. In particular, in the first paragraph of 5.6, we state precisely what we need to do to show that V_n is a fine moduli space.

3. SAGBI error fixed. Replaced with: "The normal form monomials are not the set of monomials outside some monomial initial ideal, because a normal monomial m may have a factor m' such that m' is not normal. It might be worthwhile in the future to investigate what term orders are well-suited to the study of the combinatorial properties of these toric varieties."

4. Added official definition of "Kempe embedding" and gave reference to it. See below for official definition appearing in in Part I, section 2, below Kempe's theorem.

The Kempe embedding Since the X_Γ for $\deg(\Gamma) = \mathbf{w}$ generate the algebra $R_{\mathbf{w}}$, we shall use the X_Γ to define an embedding of $M_{\mathbf{w}}$ into projective space. We dub this embedding the *Kempe embedding*.

5. changed "results" to "result".

6. changed to "... for the rank 14 space of quadric relations."

7. "analagous" changed to "analogous". There is no more reference to "Step 5".

8. clarified that the computer calculation for $n = 10$ that the quadrics generate the ideal was over $\mathbb{Q}$. We did not check this result over $\mathbb{Z}$ or $\mathbb{Z}[1/3]$.

9. changed to:

There is a bijection between the components of $X_\Gamma = 0$ and those $j < k$ such that $w_j + w_k < \sum w_i/2$, where the component D_{jk} corresponding to (j, k) is isomorphic to $M_{\mathbf{w}'}$, where $\mathbf{w}'$ is the same as $\mathbf{w}$ except w_j and w_k are removed, and $w_j + w_k$ is added. The component D_{jk} appears with multiplicity equal to the number m_{jk} of edges joining j and k in Γ.

10. (to the Referee) It really should be $n-1$ not n, since $\mathbf{w}'$ has one less component. The entries w_i and w_j were gathered together into one entry $w_i + w_j$, and so n drops to $n-1$.

11. clarified that the resulting graph G is noncrossing.

12. changed appropriate s_{ij}'s into $s_{k\ell}$'s.

13. restated the theorem into two cases, the first is for $n \geq 8$ working over $\mathbb{Z}[1/3]$ and using only quadrics, and the second is for arbitrary n over $\mathbb{Z}$ with the cubics added in. We defined the ideal with generalized Segre cubics added in to be J_V. Clarified in which parts of the proof are these assumptions used and where there is a difference between the two cases - specifically in step 1c and step 2.

14. This is fixed.

15. There are two points here. In 5.2, we use 0 and 1 as the two points of $\mathbb{P}^1$ for a strictly semistable point, while in the previous section we used 0 and ∞. That is indeed jarring, but we can't see any clean way of rectifying this, so we apologize. In the proof of 5.1, we now explicitly warn the reader that here Δ is not an $(n-6)$-matching.

16. *We couldn't understand what the referee was getting at here, and would be happy to make changes if the referee could say more.*

17. This is fixed.

18. Fixed: we (re-)describe the map right at that point.

19. changed "Remark" to "Proposition".

20. We explain why the family is stable in the last sentence of the paragraph following (a) and (b) in 5.6.

21. Fixed: "(including both X_Γ and X_Δ)" was incorrect, and now corrected to "(including X_Γ)"

22. clarified that the $\bar{x}_i$ are *algebra* generators.

23. Clarified "leftmost term":

If we always write the higher weight term to the left, then after enough Plücker relations as above have been applied so that all terms are non-crossing, the leftmost term X_G of the expansion will satisfy $w(G) = w(G_1) + w(G_2)$, and if $X_{G'}$ is any term other than the leftmost term X_G then $w(G') < w(G_1) + w(G_2)$.

24. Reference for Gelfand-Tsetlin patterns added

(Gel'fand Tsetlin pattern polytopes are studied for example in DeLoeraMcAllister).

25. Changed $\star$ to $\diamond$. The proof is also extended so as to clarify the uniqueness of G.