GRADUATE STUDENT WARM-UP WORKSHOP (ALGEBRAIC GEOMETRY BOOT CAMP) LECTURES AND ABSTRACTS

Introduction to Chevalley groups (Daniel Allcock, Texas)

I'll explain Chevalley's construction of the analogues of Lie groups, including the exceptional ones like E8, over arbitrary fields. His construction is beautiful and uniform and highlights the most important part of the structure theory of algebraic groups. By taking the field to be finite we get finite groups, and these groups account for most of the finite simple groups.

What's the deal with Geometric Langlands? (David Ben-Zvi, Texas)

The Geometric Langlands Program is one of the topics for Week 3, and the subject of a great amount of (largely inscrutable) recent activity. I'll give an overview of the program and focus on some of the key geometric players (which also make other appearances in the Seattle curriculum), including moduli stacks of bundles, derived categories of sheaves and Fourier-Mukai transforms.

What sorts of algebro-geometric objects arise in physics and why should we care? (Jim Bryan, British Columbia)

There has been a lot of excitement in the last few decades over the interaction between algebraic geometry and theoretical physics. Much of it can be bewildering to students and researchers alike as they try to navigate physics jargon and what often seems like strangely motivated constructions and questions. In this talk I will give a users guide to understanding some of the questions, objects, and ideas that arise from modern theoretical physics.

Equivariant cohomology (Linda Chen, Ohio State)

Given a group action on a variety, we obtain an equivariant cohomology ring. While the equivariant cohomology ring is larger than the ordinary cohomology ring, it is sometimes easier to perform computations in this larger ring. I will give some concrete examples and some methods of computation.

From strings to standard models via algebraic geometry (Ron Donagi, Penn)

Algebraic geometry is a crucial tool for many questions arising from string theory. In the lecture I will focus on one such question, that of deriving the real world ("the Standard Model") from string theory, and specifically from heterotic string theory. This leads

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to various questions on moduli spaces of Calabi-Yau threefolds together with stable bundles on them. We will explore some of the issues involved, especially when the Calabi-Yau has either an elliptic fibration (leading to Grand Unified theories) or a genus one fibration (leading to Standard-Model-like theories). The geometric tools include techniques for construction of Calabi-Yau manifolds (toric geometry, elliptic fibrations, orbifolds); construction of interesting vector bundles on a given manifold (Fourier-Mukai transforms, monads); and explicit calculation of various cohomology groups and related invariants.

Syzygies of algebraic varieties and Green's Conjecture (Gavril Farkas, Texas)

Syzygies of rings are classical objects which originate in Hilbert's work in Invariant Theory. In the case of coordinate rings of projective varieties, their study measures the interaction between the intrinsic geometry of a variety and its defining equations in a projective embedding.

Green's Conjecture on syzygies of a canonically embedded curve is a vast generalization of classical results from the theory of algebraic curves and loosely speaking says that one can recover the most interesting intrinsic invariant of a curve from the equations of its canonical image. Green's Conjecture has been one of the defining problems in the field for the last couple of decades and recently Voisin has found a beautiful solution in the case of the general curve of genus g.

In this talk I will introduce the problem, discuss a number of classical examples and describe several algebro-geometric ways of studying syzygies.

What is a moduli space and why do algebraic geometers love them? (Angela Gibney, Penn)

An important theme in modern mathematics is to study objects (e.g. smooth curves, vector bundles on a fixed variety, etc.) as they vary in families. When particularly lucky, one may construct a space which parametrizes the collection of all such objects in a nice way. Such a space is called a moduli space. In this talk I will give examples of moduli spaces and illustrate their usefulness in algebraic geometry.

Introduction to stacks (Tom Graber, Berkeley)

While it is now acknowledged that algebraic stacks should play a fundamental role in the theory of moduli, their study is made difficult both by the lack of any introductory reference and by a reputation for technical difficulty. I'll try to explain what a stack is, and what are some of the ways that stacks are like or unlike schemes in the context of simple examples.

Density of rational points (Brendan Hassett, Rice)

The Mordell conjecture (as proved by Faltings over number fields and Manin over function fields) insures that rational points are sparse for curves of genus > 1. On the other hand, they are plentiful on curves of genus 0 or 1, at least after a finite extension of the ground field. For example, $x^2 + y^2 = -1$ has many solutions over $\mathbb{Q}(i)$.

It is expected that this dichotomy should persist in higher dimensions. Lang has conjectured that rational points on varieties with positive canonical divisor should be contained in some proper subvariety. On the other hand, when the canonical class is negative rational points should be Zariski dense, at least after finite extension (in which case they are said to be 'potentially dense'.) For rationally connected varieties over the function field of a complex curve, Kollár-Miyaoka-Mori and Graber-Harris-Starr have shown that rational points are dense, even without taking extensions. In this case, rational points satisfy quite strong approximation properties, e.g., there exists rational points with prescribed reductions modulo primes.

When the canonical class is trivial, it is less clear what to expect. Abelian varieties have potentially dense rational points, and large classes of K3 surfaces do as well. However, for most K3 surfaces potential density remains an open problem.

Why rational curves? (Stefan Kebekus, Cologne)

One approach to investigate the structure of an algebraic variety X is to study the geometry of curves, especially the rational curves, that X contains. This approach relies on classical geometric ideas and strives to understand the intrinsic geometry of varieties. It is nowadays understood that if X contains many rational curves, then their geometry determines X to a large degree.

After Shigefumi Mori showed that many interesting varieties contain rational curves, their systematic study became a standard tool in algebraic geometry. The spectrum of application is diverse and covers long-standing problems such as deformation rigidity, stability of the tangent bundle, classification problems, and questions concerning the existence of non-trivial finite morphisms between Fano manifolds.

The expository lecture concentrates on examples and basic properties of minimal degree rational curves on projective varieties. Some of the more advanced applications will be briefly discussed.

Langlands' conjectures and Shimura varieties (Elena Mantovan, Berkeley)

In 1967, Langlands suggested the existence of a relation between two seemingly unrelated mathematical objects: Galois representations and automorphic representations. Since then, the work of many mathematicians has focused on isolating and constructing algebraic varieties whose geometry is supposed to explain the existence of such correspondences.

For correspondences defined over a number field, this role is played by Shimura varieties. In my talk I will discuss some aspects of Langlands' conjectures and how they are reflected in the geometry of the Shimura varieties.

Birational Classification of Varieties (James McKernan, Santa Barbara)

This talk will give an introduction to some of the main ideas of higher dimensional geometry and the minimal model program. Our picture of the classification of higher dimensional varieties is largely shaped by the birational classification of curves and surfaces. Two main features arise; the birational geometry of a variety is strongly controlled by the rational curves on the variety and the behaviour of the canonical divisor.

Combinatorial positivity in algebraic geometry (Ezra Miller, Minnesota)

Combinatorics can be a great excuse to talk about really cool properties of familiar spaces from algebraic geometry. These spaces run the gamut from flag varieties and degeneracy loci for vector bundle morphisms to toric varieties and Hilbert schemes. The combinatorics can often be phrased as the question, "You've just handed me a certain polynomial; why do its coefficients seem to be positive integers?" The algebraic geometry can involve calculations in various kinds of cohomology and K-theory accompanied by various kinds of flat degenerations.

Non-exhaustive list of examples, some of which we could study during the workshop:

(1) Haiman's positivity proof for the coefficients of Macdonald polynomials using the K-theory of Hilbert schemes of points in the plane.
(2) Knutson-Miller-Shimozono degenerations of matrix Schubert varieties and quiver loci to explain the positivity of Schubert polynomials and prove positivity for the Buch-Fulton quiver polynomials in Chow cohomology.
(3) Speyer's degeneration of the apiary ("triple flag") variety to the hive toric variety to explain geometrically the Knutson-Tao honeycomb description of Littlewood-Richardson numbers.
(4) Related to number 3: the Gonciulea-Lakshmibai degeneration of the ordinary flag variety to the Gelfand-Tsetlin toric variety.
(5) Vakil's geometric Littlewood-Richardson rule by successively degenerating intersections of Schubert varieties.

Spaces of arcs and birational geometry (Mircea Mustata, Michigan)

The space of arcs of a smooth variety is the set of all morphisms from the formal disk to the given variety. This is an infinite-dimensional space that turns out to be very useful also in studying finite-dimensional phenomena. I will give an introduction to spaces of arcs, explaining their relevance in studying birational maps. In particular, we will see why these spaces are useful for defining invariants of singular varieties and for encoding properties of singularities.

Hilbert schemes (Mike Roth, Queens)

The notion of a parameter or moduli space is one of the key mathematical ideas of the 20th century. In algebraic geometry, Hilbert Schemes are the basic example of such a parameter space, and serve as a starting place for many constructions of other parameter spaces, including the moduli space of curves.

The point of the talk is to go over what we mean by a parameter or moduli space, what the Hilbert scheme parameterizes, and how to construct it.