There is a point of view on twisting by n that has always made more sense to me than the usual definition (as well as helping me keep track of indices). Let $X = \operatorname{Proj} S$. The usual Proj construction gives us a sheaf of rings $\mathcal{O}_X$ on X; however, with a slight modification of this construction, we obtain instead a sheaf of graded rings

$$\mathcal{S} = \bigoplus_{n\in\mathbb{Z}} \mathcal{O}_X(n).$$

This can be used as the definition for the twisting sheaves $\mathcal{O}_X(n)$. A more precise explanation follows.

Let $S_\bullet$ be a $\mathbb{Z}^+$-graded ring, finitely generated over $A = S_0$ and generated in degree one. Let $X = \operatorname{Proj} S_\bullet$.

Let M be a graded S-module. Then

$$\Gamma(D_+(f), \mathcal{M}) = M_f$$

defines a quasicoherent sheaf $\mathcal{M}$ of graded $\mathcal{O}_X$-modules. We then have

$$\widetilde{M_+} = \mathcal{M}_0$$
$$\widetilde{M_+}(n) = \mathcal{M}_n.$$

It is easy to define a natural map $M \to \Gamma(X, \mathcal{M})$. Since this respects grading, we obtain maps $M_n \to \Gamma(X, \widetilde{M_+}(n))$.

In the particular case that $M = S$, we have that $\mathcal{S}$ is in fact the graded $\mathcal{O}_X$-algebra

$$\bigoplus_{n\in\mathbb{Z}} \mathcal{O}_X(n).$$

If S is a polynomial ring over A, so that $X = \mathbb{P}^n_A$, then it is not hard to show that the natural maps $S_n \to \Gamma(X, \mathcal{S}_n)$ are isomorphisms, telling us precisely the global sections of $\mathcal{O}(n)$ in this case. One can do something similar to equate rational sections with homogeneous rational functions in this case.

One shows that

$$\mathcal{S}_n \otimes \widetilde{M_+} \to \widetilde{M_+}(n)$$

is an isomorphism by restricting to $D_+(f)$, where f is of degree one. These cover X since S is generated in degree one. Thus, for a general quasicoherent sheaf $\mathcal{F}$, we may define $\mathcal{F}(n) = \mathcal{O}_X(n) \otimes \mathcal{F}$, as usual. This shows in particular that the notion of twisting by n as defined above is well-defined on quasicoherent sheaves.