18.03 Problem Set 6.5

This is another set of problems not to be handed in, just to give you practice for the Hour Exam on Wednesday, April 21.

Syllabus

23. (F 2 Apr) Complex or repeated eigenvalues: Notes LS 1.3, 1.4.

24. (M 5 Apr) Dynamics of linear autonomous systems: Notes GS 1–5, EP 7.3 (522–530), Portrait Gallery Handout.

25. (W 7 Apr) Dynamics of nonlinear autonomous systems; linearization: Notes GS 6–7.

26. (F 9 Apr) Example: the nonlinear pendulum: EP 7.5 (561).

27. (M 12 Apr) Initial Value problems: Notes LS 2.1–2.3, EP 5.7 (443–447).

28. (W 14 Apr) Matrix exponentials: LS 2.4, EP 5.8 (454–457; the series expression for the exponential is not essential).

29. (F 16 Apr) Spinning books—a review: Handout on Euler's equations.

30. (W 21 Apr) Hour Exam III

Part I.

27. (M 12 Apr) EP 5.7: 15, 17, 19.

28. (W 14 Apr) EP 5.8: 1, 3; Notes LS 32, 36.

29. (F 16 Apr) TBA

Part II.

27. (M 12 Apr) Suppose $\ddot{x} + e^x = 0$. Write down the associated system of first order equations, using $y = \dot{x}$. Then find the "first integral" of this autonomous system, by thinking of y as a function of x, verifying that $yy' = \dot{y}$ (where $y' = dy/dx, \dot{y} = dy/dt$), and solving the resulting first order ODE for y as a function of x. You have the Matlab tools by now to cause it to plot the resulting functions $y(x)$ and to plot the trajectories of the system, and you may feel like checking your work.

28. (W 14 Apr) (a) Verify that multiplication by the complex number $a + bi$ is represented by the matrix $A(a + bi) = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, in the sense that if $(a + bi)(x + yi) = p + qi$ then $A(a + bi) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$. Check that $A(z)A(w) = A(zw)$ and $A(z + w) = A(z) + A(w)$. Convince yourself that $A(\cos\theta + i\sin\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ has the effect of rotating counterclockwise by an angle θ, and that (for r real) $A(r) = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$

has the effect of multiplying vectors by r. What are the eigenvalues of $A(z)$? What are the eigenvectors? What is $e^{A(z)}$? Does this help convince you that the expression $e^{a+bi} = e^a(\cos b + i \sin b)$ is reasonable?

(b) Answer Notes LS 37, by the following method: the first column of e^{At} is the solution to the IVP $\dot{x} = Ax$ with initial condition $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and the second column is the solution with initial condition $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find these solutions.

29. (F 16 Apr) Use `ode45` and `plot` to get a better idea of how the axis moves according to the Euler equations ((7) in the handout on them) with $\lambda_1 = 1, \lambda_2 = 10, \lambda_3 = 100$ and initial conditions $l_1(0) = 1, l_2(0) = 2, l_3(0) = 3$. Take the time interval to be 20. You might want to see what the trajectory swept out by the axis of rotation in three-space looks like, using `plot3` (about which you can find some information using `help plot3`).