# 18.03 Problem Set 6.5

This is another set of problems not to be handed in, just to give you practice for the Hour Exam on Wednesday, April 21.

**Syllabus**

**23. (F 2 Apr)** Complex or repeated eigenvalues: Notes LS 1.3, 1.4.

**24. (M 5 Apr)** Dynamics of linear autonomous systems: Notes GS 1–5, EP 7.3 (522–530), Portrait Gallery Handout.

**25. (W 7 Apr)** Dynamics of nonlinear autonomous systems; linearization: Notes GS 6–7.

**26. (F 9 Apr)** Example: the nonlinear pendulum: EP 7.5 (561).

**27. (M 12 Apr)** Initial Value problems: Notes LS 2.1–2.3, EP 5.7 (443–447).

**28. (W 14 Apr)** Matrix exponentials: LS 2.4, EP 5.8 (454–457; the series expression for the exponential is not essential).

**29. (F 16 Apr)** Spinning books—a review: Handout on Euler's equations.

**30. (W 21 Apr) Hour Exam III**

**Part I.**

**27. (M 12 Apr)** EP 5.7: 15, 17, 19.

**28. (W 14 Apr)** EP 5.8: 1, 3; Notes LS 32, 36.

**29. (F 16 Apr)** TBA

**Part II.**

**27. (M 12 Apr)** Suppose $\ddot{x} + e^x = 0$. Write down the associated system of first order equations, using $y = \dot{x}$. Then find the "first integral" of this autonomous system, by thinking of $y$ as a function of $x$, verifying that $yy' = \dot{y}$ (where $y' = dy/dx, \dot{y} = dy/dt$), and solving the resulting first order ODE for $y$ as a function of $x$. You have the MATLAB tools by now to cause it to plot the resulting functions $y(x)$ and to plot the trajectories of the system, and you may feel like checking your work.

**28. (W 14 Apr) (a)** Verify that multiplication by the complex number $a + bi$ is represented by the matrix $A(a+bi) = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, in the sense that if $(a+bi)(x+yi) = p + qi$ then $A(a+bi)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$. Check that $A(z)A(w) = A(zw)$ and $A(z+w) = A(z) + A(w)$. Convince yourself that $A(\cos\theta + i\sin\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ has the effect of rotating counterclockwise by an angle $\theta$, and that (for $r$ real) $A(r) = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$

has the effect of multiplying vectors by $r$. What are the eigenvalues of $A(z)$? What are the eigenvectors? What is $e^{A(z)}$? Does this help convince you that the expression $e^{a+bi} = e^a(\cos b + i \sin b)$ is reasonable?

**(b)** Answer Notes LS 37, by the following method: the first column of $e^{At}$ is the solution to the IVP $\dot{x} = Ax$ with initial condition $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and the second column is the solution with initial condition $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find these solutions.

**29. (F 16 Apr)** Use `ode45` and `plot` to get a better idea of how the axis moves according to the Euler equations ((7) in the handout on them) with $\lambda_1 = 1, \lambda_2 = 10, \lambda_3 = 100$ and initial conditions $l_1(0) = 1, l_2(0) = 2, l_3(0) = 3$. Take the time interval to be 20. You might want to see what the trajectory swept out by the axis of rotation in three-space looks like, using `plot3` (about which you can find some information using `help plot3`).