18.03 Problem Set 4.5

These problems are not to be turned in. They are just to keep you on your toes and off the street during the space of time before Wednesday's Hour Exam.

Syllabus

17. (F 12 Mar) Nonconstant coefficients, reduction of order, nonlinear examples: EP 2.6, handout on the Airy equation.

18. (T 15 Mar) Review.

19. (W 17 Mar) Hour Exam II.

Part I.

17. (F 12 Mar) EP 2.6: 19 (use the method from problem **12.**; 23, 27, 31.

18. (M 15 Mar) A practice hour exam will be handed out on Monday.

Part II.

17. (F 12 Mar) (a) In **12.** you obtained "Abel's equation," which said that given one solution. y_1, to a homogeneous linear second order equation, $y'' + p(x)y' + q(x) = 0$, you can find a second solution, y_2, by solving the *first order inhomogeneous* linear equation

$$y_1 y_2' - y_1' y_2 = e^{-\int p(x)dx}.$$

Use an integrating factor to explain why this leads precisely to the reduction of order formula, EP 2.6 (8).

(b) Here's an operator-theoretic approach to Euler-Cauchy equations. xD is an operator on functions of x: $(xD)y = xy'$. Show that $x^2D^2 = (xD)^2 - xD$. Thus the Euler-Cauchy operator

$$L = x^2D^2 + pxD + qI = (xD)^2 + (p-1)xD + qI = f(xD), \quad \text{where } f(r) = r^2 + (p-1)r + q.$$

Now $xDx^r = rx^r$: x^r plays the same role for xD that e^{rx} did for D, it's an eigenfunction with eigenvalue r. Thus

$$Lx^r = f(xD)x^r = f(r)x^r,$$

and if a is a root of f then x^a is a solution to the homogeneous equation $Ly = 0$.

18. Nothing new.