18.03 Practice Hour Exam III
Spring, 1999

Coverage: EP 57 (443–447), 5.8 (454–457; power series excluded), 7.3 (522–530), 7.5 (561); Notes LS (power series excluded), GS; Handout on phase portraits.

1. Let $A = \begin{bmatrix} 3 & -4 \\ 6 & -7 \end{bmatrix}$.

(a) Compute the eigenvalues of A.

(b) For each eigenvalue find a nonzero eigenvector.

(c) Circle all the adjectives below which correctly describe the phase portrait of the autonomous system $\dot{\vec{x}} = A\vec{x}$. Then draw a little picture of the phase portrait. Mark the eigenvectors.

spiral	unstable
center	neutrally stable
improper node	asymptotically stable
star node	structurally stable
proper node	structurally unstable
saddle	

2. Suppose A is a matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$ and corresponding eigenvectors $\vec{\alpha}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{\alpha}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(a) Write down a fundamental matrix $\Phi(t)$ and then normalize it to obtain e^{At}.

(b) Write down the solution to the differential equation $\dot{\vec{x}} = A\vec{x}$ such that $\vec{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) Write down the matrix A.

3. This problem concerns the nonlinear autonomous system

$$\begin{cases} \dot{x} &= y - x^2 \\ \dot{y} &= x - y^2 \end{cases}.$$

(a) How do you know that $(0,0)$ and $(1,1)$ are critical points and that there are no others?

(b) Linearize the equation at $(0,0)$ by dropping higher order terms, and identify the type (spiral, node, or saddle) and stability (stable or unstable) of this critical point.

(c) Compute the Jacobian and evaluate it at the critical point $(1,1)$.

(d) Sketch the phase portrait of this system.

4. Convert the second order ODE $x'' + 2x' + 2x = 0$ into a system and construct its phase portrait.