18.03 Practice Hour Exam II

Coverage: EP 2.1–2.6, 2.8; Handout on Undetermined Coefficients; Notes C.

1. Find: **(a)** $(1+\sqrt{3}i)^6$; **(b)** all complex numbers z such that $e^z = 2$.

2. Find the general solution of the following equations.

(a) $y'' - 3y' + 2y = e^x$.

(b) $y'' - 2y' + y = x^2 e^x$.

(c) $y'' + y = e^x \sin x$.

(d) $x^2 y - xy' - y = 0$. What is the Wronskian of your independent pair of solutions? How can you determine this (up to constant) without even finding an independent pair of solutions?

3. **(a)** Describe the various types of solution you obtain as the spring constant q varies in the equation $y'' + 4y' + qy = 0$.

(b) What is the steady state solution of $y'' + 3y' + 2y = \cos(\omega_0 x)$? For what value of ω_0 is the gain maximized? What is the phase lag as a function of ω_0?

4. **(a)** Which of the following operators are linear? $x^2 D$, $Sy = y(\sin(x))$, $Ty = y + 1$, $Ly = x^2 Dy + \sin(x)y$, $M_f y = f(x)y$.

(b) Express the linear differential operator $(xD)^3$ in more standard terms.

(c) Explain why $f(xD)x^r = f(r)x^r$, where f is a polynomial, and why this amounts to the Euler-Cauchy story.

5. For what values of p, q is there a solution of $y'' + py' + qy = 0$ which is not zero but which converges to zero as $x \to \infty$? For which values do all nonzero solutions have this property? What does this have to do with the roots of the characteristic polynomial?

6. EP 2.6: 37, 41, 43, 46, 47.