

## 18.03 Practice Hour Exam II

Coverage: EP 2.1–2.6, 2.8; Handout on Undetermined Coefficients; Notes C.

1. Find: (a)  $(1 + \sqrt{3}i)^6$ ; (b) all complex numbers  $z$  such that  $e^z = 2$ .

2. Find the general solution of the following equations.

(a)  $y'' - 3y' + 2y = e^x$ .

(b)  $y'' - 2y' + y = x^2e^x$ .

(c)  $y'' + y = e^x \sin x$ .

(d)  $x^2y - xy' - y = 0$ . What is the Wronskian of your independent pair of solutions? How can you determine this (up to constant) without even finding an independent pair of solutions?

3. (a) Describe the various types of solution you obtain as the spring constant  $q$  varies in the equation  $y'' + 4y' + qy = 0$ .

(b) What is the steady state solution of  $y'' + 3y' + 2y = \cos(\omega_0x)$ ? For what value of  $\omega_0$  is the gain maximized? What is the phase lag as a function of  $\omega_0$ ?

4. (a) Which of the following operators are linear?  $x^2D$ ,  $Sy = y(\sin(x))$ ,  $Ty = y + 1$ ,  $Ly = x^2Dy + \sin(x)y$ ,  $M_fy = f(x)y$ .

(b) Express the linear differential operator  $(xD)^3$  in more standard terms.

(c) Explain why  $f(xD)x^r = f(r)x^r$ , where  $f$  is a polynomial, and why this amounts to the Euler-Cauchy story.

5. For what values of  $p, q$  is there a solution of  $y'' + py' + qy = 0$  which is not zero but which converges to zero as  $x \rightarrow \infty$ ? For which values do all nonzero solutions have this property? What does this have to do with the roots of the characteristic polynomial?

6. EP 2.6: 37, 41, 43, 46, 47.