18.03 Class 1, Feb 3, 1999

Introduction. What are differential equations; how they arise; what is a solution to a DE; types of solution (particular, general, implicit); initial conditions; separable equations; orthogonal families.

I'll try to draw the analogy with an algebraic equation, which typically has one or at worst a finite number of *numbers* as solutions. Meaning of "ordinary" and "order."

The simplest is $y' = f(x)$. Its solution is the subject of "integral calculus," and much of 18.03 is devoted to reducing more complicated DEs to this one so as to use integration.

I'll give a simple example ($f(x) = x^2$); note the constant of integration, and that you get a new one for every successive integration, so an n-th order equation should have an n-parameter family of solutions. This is the *general* solution; a *particular* solution is typically picked out by specifying an *initial condition.*

Differential equiations come mostly from scientific models. The science tells us only how a system changes "from one instant to the next." This is the DE. The mathematical discipline of Differential Equations *amplifies* this information, giving an analytic or numerical description of the *global* behavior of the system. Of special interest is the long-term behavior. Once the math is done, the science reenters to *interpret* this information.

Within mathematics DEs often arise from geometry. For example: what is the increasing curve through $(0, 1)$ with the property that for each (x, y) on the curve the the area of the triangle formed by the x-axis, the tangent line to the curve at (x, y), and the vertical line through (x, y), is equal to 1. This leads to the DE $y' = y^2/2$.

I'll solve this DE by *separating variables.* Notice that the *two* constants of integration can be merged to *one*. The resulting curves are *horizontal* translates of one another, as must be the case since the location of the y-axis does not enter into the statement of the problem. The particular solution containing $(0, 1)$ is $y = 2/(2 - x)$. Note also that the solutions of this DE do not extend over the whole range of values of x for which the DE looks nice: a reflection of the fact that it is a *nonlinear* DE.

A first order DE gives rise to a family of curves, and, conversely. For example $y = ce^{-x}$ satisfies $y' = -y$ for any c. The *orthogonal* family of curves satisfies $y' = 1/y$, which has as solution curves the parabolas $x = y^2/2 + c$. While we could solve here for y in terms of x (at the expense of restricting to the top or bottom half of each parabola), it's more natural to leave the solution as is. More generally, a solution may appear as simply a *relation* between x and y; this is an *implicit* solution.