ALGEBRAIC GEOMETRY (MATH 145) PROBLEM SET 6

This set is due by 4pm on Friday, February 24, in Laurent Côté's mailbox.

You are encouraged to talk to each other (in person, privately, on piazza, over pizza, etc.), and to me, about the problems. But write up your solutions separately, and also give credit for ideas and sources. If you don't know people, but would like to work together, just let me know, and I'll introduce you to others. Some of these problems require hints, and I'm happy to give them!

You are allowed to assume the answer to one problem in solving another (within reason — if in doubt, just ask).

Questions? Try asking on piazza (this is the first time I'm using it in a class, and so far I like it), and also feel free to email me or Laurent.

Reminder: My office hours are Wednesdays 9:15-11:15 am and Fridays 2:30-3:30 pm in 383-Q. Laurent's office hours are Wednesdays 3:30-4:15 pm and Thursdays 7-8:15 pm in 381-L.

0. (mandatory) Complete the online check-in on canvas (by the due date).

Also, do **four** of the following problems. If you are ambitious (and have the time), go for more. *You may assume Hilbert's Nullstellensatz in solving these problems. Remember that the Nullstellensatz has the hypothesis that the "base field" k is algebraically closed!*

In many of these problems, I'm assuming the base field k is algebraically closed to avoid distracting you. Sometimes this hypothesis is essential, and sometimes it is not.

1. Suppose R is a finitely generated algebra over an algebraically closed field k, that is an integral domain, and suppose $f \in R \setminus 0$. Show that the sections of the sheaf of algebraic functions $\mathcal{O}_{\mathfrak{m}\operatorname{Spec} R}$ over the distinguished open set $D(f)$ are precisely R_f.

2. Suppose R is a finitely generated algebra over an algebraically closed field k, that is an integral domain, and $p = [\mathfrak{m}]$ is a point of $\mathfrak{m}\operatorname{Spec} R$. Show that the stalk of $\mathcal{O}_{\operatorname{Spec} R}$ at the point p is the local ring $R_{\mathfrak{m}}$.

3. Suppose R is a finitely generated algebra over an algebraically closed field, and $U \subset \mathfrak{m}\operatorname{Spec} R$ is an open subset. Show that there are finitely many elements $f_1, \ldots, f_n$ of R such that $U = \cup_{i=1}^{n} D(f_i)$.

4. Suppose R is a ring, and $f \in R$. Show that the prime ideals of R_f can be identified with those prime ideals of R which do not contain f. If R is finitely generated over an

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algebraically closed field k, show that that the maximal ideals of R_f can be identified with those maximal ideals of R which do not contain f, i.e., the points of $D(f)$.

5. Suppose R is a ring, and $S \subset R$ is a multiplicative subset (if you know what this is). Show that the prime ideals of the localization $S^{-1}R$ can be identified with those prime ideals of R containing no elements of S. If R is finitely generated over an algebraically closed field k, show that that the maximal ideals of $S^{-1}R$ can be identified with those maximal ideals of R containing no elements of S.

6. Show that the only (algebraic) functions on the algebraic variety $\mathbb{P}^3_k$ are the constant functions (i.e., k).

7. Sketch out your plan for your writing project, in latex, spelling out the things you intend to do to fill it in. (Perhaps: make an overleaf document that you can share with Laurent.)

8. Write a rough draft of your writing project, in latex. (Perhaps: make an overleaf document that you can share with Laurent.) If you do this, you also get the points for problem 7.

E-mail address: `vakil@math.stanford.edu`