We first construct a map \( \text{Coh}(X) \to K_0(X) \) of locally free \( K \)-sheaves. Check: kernels of the exact sequences agree.

\[
\begin{equation}
\begin{array}{c}
\longrightarrow K \\
\downarrow \\
 K' = K' \\
\downarrow \\
\longrightarrow P_x P' \\
\downarrow \\
 P_0 \\
\downarrow \\
 0 \\
\downarrow \\
0 \\
\end{array}
\end{equation}
\]

\( P'' \to P_x P_0 = \{ (m, n) \in P_0 \otimes P_0' \mid m = n \} \in A ? \).

\[
\begin{equation}
\begin{array}{c}
\longrightarrow K \\
\downarrow \\
 p'' \\
\downarrow \\
 P_0 \\
\downarrow \\
 0 \\
\downarrow \\
0 \\
\end{array}
\end{equation}
\]

\( K_0 \) is locally free, since \( P'' \to P' \) is surjective (and \( K_0 \) is locally free sheaves).

\[
\begin{equation}
\begin{array}{c}
\longrightarrow Q \\
\downarrow \\
 K_0 \\
\downarrow \\
 K' \\
\downarrow \\
0 \\
\end{array}
\end{equation}
\]

We have two resolutions of \( K^0, K' \):

\[
\begin{equation}
\begin{array}{c}
P_0, \to K^0 \\
\downarrow \\
K_0, \to K' \\
\end{array}
\end{equation}
\]

and

\[
\begin{equation}
\begin{array}{c}
P_0, \to K^0 \\
\downarrow \\
K_0, \to K' \\
\end{array}
\end{equation}
\]

Check: \( \text{Ker} K_0 \to K \), \( \text{Ker} K_0 \to K' \), whose kernels agree.

\[
\begin{equation}
\begin{array}{c}
P_0, \to K^0 \\
\downarrow \\
K_0, \to K' \\
\end{array}
\end{equation}
\]

and two resolutions of \( Q \):

\[
\begin{equation}
\begin{array}{c}
K_0, \to K \\
\downarrow \\
K_0, \to K' \\
\end{array}
\end{equation}
\]
By induction on lengths of locally free resolutions,
(by general results we should be able to ensure
length of $K_0$ is length of $P_{\nu +1}$)
We may assume that these resolutions give the
same objects in locally free $K$-theory.
Thus $[K_I, K'_I, \{Q\}]$ are defined in $K$-theory,
and we have the relations
$[K_I] = [K_0] - [\{Q\}]$
$[K'_I] = [K'_0] - [\{Q\}]$
Using resolutions $K_0 \to K$
and $K'_0 \to Q$,
Using the two SES of $\text{loc. Free}$ in the "cross" of
the above diagram, we obtain
$[K_0] + [P_0] = [P'] = [K'_0] + [P'_0]$.
Thus
$E(R) \times \mathbb{Z} \to \mathbb{Z}$
$[P_0] - [K_I] = [P'_0] - [K'_I]$.
Using the resolutions $P_{\nu+1} \to K$
we obtain
$P_0 \to K'$
$\Sigma(-1)^i [P_i] = \Sigma(-1)^i [P'_i]$.
Thus $\text{Coh}(X) \to K_0(K)$ is well defined.

Note that we proved, along the way, that
given
$0 \to C \to P \to A \to 0$ w/ $P$ \text{loc. Free},
$[A] + [C] = [P] \in K_0(X)$.

For more general SES of \text{coh} sheaves,
$0 \to C \to B \to A \to 0$,
and a \text{proj} \text{loc. Free} $P \to A$,
the same "projective over fiber product" trick
shows $\{Q \text{ loc. Free}\}$

st. $0 \to C \to B \to A \to 0$ holds.
Thus, we have proved that
\[ \text{coast}(X) \Rightarrow \text{cost}(X) \]

This implies
\[ [CT,T] = [T] \]
and we have proven, since \( P \subseteq Q \subseteq \tilde{R} \) (as defined),
that the existence also holds in the columns.

Thus, as before, \( R \) is locally free, and this diagram exists.