MATH 210A PROBLEM SET 5

Do 7 of the following 8 problems. This problem set will be due on Thursday November 20 at noon in Evita Nestoridi's mailbox on the first floor of the mathematics department, or in class. If and when you find typos or errors on this or any other problem set, please let me know, so I can warn others.

1. Suppose G_1, G_2, $\ldots$, are finite abelian groups acting on a finite-dimensional vector space V over $\mathbb{C}$. Suppose further that their actions commute. Show that their action can be simultaneously diagonalized, i.e. that there is a basis of V under which all the actions are diagonal.

2. Suppose T is an endomorphism of a finite-dimensional vector space over a field k, and that the minimal polynomial of T is divisible by t but not by t^2. Show that there is a polynomial in T that is a projection onto the kernel of T. Show that if t^2 is a factor of T, then there is *no* such polynomial.

3. Suppose V is the "subpermutation" 3-dimensional irreducible representation of S_4, over a characteristic 0 field. Decompose $\operatorname{Sym}^2 V$ into irreducible representations of S_4. More precisely, find the multiplicity with which each irreducible representation of S_4 appears in $\operatorname{Sym}^2 V$.

4. Suppose G is a finite group, and k an *algebraically closed* field with characteristic not dividing $|G|$. Suppose V is an irreducible representation of G (over k). If $\rho : G \to GL(W)$ is a representation of G on a finite-dimensional vector space W, show that

$$\frac{\dim |V|}{|G|} \sum_{g \in G} \chi_V(g^{-1})\rho(g) : W \to W$$

is a morphism of G-representations (i.e., of kG-modules), which is projection onto the "V-isotypic" part of of W (sending any irreducible subrepresentation of W not isomorphic to V to 0, and acting as the identity on every irreducible subrepresentation of W isomorphic to V). *Try not to use Wedderburn's Theorem if possible, or results from the text whose proofs rely on it!*

5. DF 18.3.7.

6. DF 18.3.8.

7. DF 18.3.10.

8. DF 18.3.11.

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