MATH 120 PRACTICE MIDTERM

1. (6 points)
   (a) What is the order of $A_4$?
   (b) How many rotations of the cube have order exactly 2 (i.e. if you do them twice, you get the identity, but they are not the identity)? Possible hint: we have seen that the group of rotations of the cube is isomorphic to $S_4$.
   (c) Which of the following is true?
      (i) For all subsets $A$ of a group $G$, the centralizer $C_G(A)$ is always contained in the normalizer $N_G(A)$.
      (ii) For all subsets $A$ of a group $G$, the centralizer $C_G(A)$ always contains in the normalizer $N_G(A)$.
      (iii) Neither (i) nor (ii) is true.

2. (6 points) Suppose $G$ acts on a set $A$, and $a$ and $b \in A$ are in the same orbit of $G$. Show that $G_a$, the stabilizer of $a$, is conjugate to $G_b$, the stabilizer of $b$.

3. (6 points) Suppose $H \leq K \leq G$. Prove that $|G : H| = |G : K| \cdot |K : H|$. (Do not assume the groups are finite!)

4. (6 points) Let $G$ be any group.
   (a) Prove that the map $G \rightarrow G$ defined by $g \mapsto g^2$ is a homomorphism if and only if $G$ is abelian.
   (b) If $G$ is abelian and finite show that this map is an isomorphism if and only if $G$ has odd order.

5. (6 points) Suppose $G$ is group of order $p^2q$, where $p$ and $q$ are distinct prime numbers. Show that $G$ is not simple. (This is in the text, so of course do not just quote the text!)

6. (6 points) Suppose that $H_1$ and $H_2$ are groups of finite index in $G$. Show that $H_1 \cap H_2$ is also of finite index in $G$. 

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