MATH 120 PRACTICE MIDTERM

Give complete proofs unless otherwise indicated. Each question is worth 6 points. They are not in order of difficulty.

1. For this question, give answers only.
   (a) Find the order of the element (12)(13)(14) in S_4.
   (b) Which of the following groups are isomorphic: D_6, S_3, Z/6?
   (c) Which of the following is true? (i) For all subsets A of a group G, the centralizer C_G(A) is always contained in the normalizer N_G(A). (ii) For all subsets A of a group G, the centralizer C_G(A) always contains in the normalizer N_G(A). (iii) Neither (i) nor (ii) is true.

2. Prove that the subgroup generated by 9 and 6 in Z/(24Z) is cyclic, and find its order.

3. Determine whether the following subsets of the group GL_n(R) are subgroups. (Here GL_n(R) is the set of invertible n \times n matrices with real entries.)
   (a) those elements with rational entries.
   (b) those elements with integer entries.

4. Suppose \phi : G \rightarrow H is a group homomorphism. Suppose H' < H is a subgroup of H. Show that \phi^{-1}(H') (those elements of G mapped by \phi into H') is a subgroup of G.

5. Show that Z/7Z, the integers modulo 7, form a field (with the usual addition and multiplication).

6. Show that there is only one group homomorphism from Q to R sending 1 to 2. (The operation for both Q and R is addition.)

Date: October 4, 2010.