MATH 120 PRACTICE MIDTERM

Give complete proofs unless otherwise indicated.

1. (6 points) For this question, give answers only.

(a) Give a Jordan-Holder decomposition of $S_3$. In other words, give a nested sequence of normal subgroups, where the quotient of each by the next smaller one is simple.

(b) What is the center of $S_3$?

2. (6 points) The group $SL(2, \mathbb{Z})$ consists of those $2 \times 2$ matrices with integer entries and determinant 1. Let $N$ be the subset consisting of matrices congruent to the identity matrices modulo 2, i.e. with odd diagonal entries and even off-diagonal entries. Show that $N$ is a normal subgroup of $SL(2, \mathbb{Z})$. (Hint: describe $N$ as the kernel of the map from $SL(2, \mathbb{Z})$ to another group.)

3. (6 points) Suppose $G$ acts on a set $A$, and $a$ and $b \in A$ are in the same orbit of $G$. Show that $G_a$, the stabilizer of $a$, is conjugate to $G_b$, the stabilizer of $b$.

4. (6 points) Describe a bijection between the conjugates of an element $g$ of a group $G$ and the cosets of the centralizer of $g$ (consisting of those elements $s$ of $G$ such that $gs = sg$).

5. (6 points) Suppose both $H$ and $K$ are normal subgroups of $G$ with $H \cap K = \{e\}$. Show that $xy = yx$ for all $x \in H$ and $y \in K$. (Hint: move everything to one side of the equation, and show that the result is $e$.)

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