MATH 120 PRACTICE MIDTERM

Write your name at the top of each page. Give complete proofs except for problem 1, where answers will suffice.

1.
(a) Is the set of rational numbers in lowest terms whose denominators are odd, along with zero, a subgroup of the rational numbers?
(b) Find the order of $(1234)(567)(89)$ in S_9.
(c) Does the rotational symmetry group of the cube act transitively on the set of vertices? Faithfully?

2. Let $H \leq K \leq G$. Prove that $|G : H| = |G : K| \cdot |K : H|$.

3. Suppose H and K are subgroups of G, and K is normal. Show that $K \cap H$ is normal in H.

4. Show that the subset $\{e, (12)(34), (13)(24), (14)(23)\}$ of S_4 is a subgroup. Show that it is normal. Show that it is contained in A_4. Show that it is normal in S_4. Show that A_4 is not simple. Find a composition series for S_4.

5. Prove that if $G/Z(G)$ is cyclic then G is abelian.

6. Suppose that H_1 and H_2 are groups of finite index in G. Show that $H_1 \cap H_2$ is also of finite index in G.

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