This problem set is due on Friday, March 16 at Jarod Alper’s office door.

1. Suppose that $A$ and $B$ are ideals with $AB \subset Q$ for a primary ideal $Q$. Prove that if $A$ is not contained in $Q$, then $B \subset \sqrt{Q}$. (Dummit and Foote 15.2 problem 29)

2. Show that the intersection of two $P$-primary ideals of a ring $R$ is also $P$-primary. (Dummit and Foote 15.2 problem 31)

3. Prove that a prime ideal $P$ contains the ideal $I$ if and only if $P$ contains one of the associated primes of a minimal primary decomposition of $I$. (Dummit and Foote 15.2 problem 37)

4. Let $P_1, \ldots, P_m$ be the associated prime ideals of the ideal $(0)$ in the Noetherian ring $R$.
   
   (a) Show that $P_1 \cap \cdots \cap P_m$ is the collection of nilpotent elements in $R$.
   
   (b) Show that $P_1 \cup \cdots \cup P_m$ is the collection of zero divisors in $R$.

   (Dummit and Foote 15.2 problem 41; some hints are given there. Caution: if you use Corollary 22 from the book, you’ll have to prove it, as we haven’t done it in class.)

5. Prove that the ideal $I$ in the Noetherian ring $R$ is radical if and only if the primary components of a minimal primary decomposition are all prime ideals, and conclude that in this case the minimal primary decomposition is unique. (Dummit and Foote 15.2 problem 43; some hints are given there.)

6. Describe the Zariski topology on $\text{Spec } \mathbb{C}[t]$.  

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*Date: Friday, March 9, 2007.*