1. Prove that if the Galois group of the splitting field of a cubic over $\mathbb{Q}$ is the cyclic group of order 3 then all the roots of the cubic are real. (Dummit and Foote p. 562, problem 13)

2. Show that $\mathbb{Q}(\sqrt{2} + \sqrt{2})$ is a cyclic quartic field, i.e. is a Galois extension of degree 4 with cyclic Galois group. (Dummit and Foote p. 562, problem 14)

3. Show that every irreducible polynomial in $\mathbb{F}_p[x]$ is a factor of $x^{p^n} - x$ for some $n$.

4. Suppose $E/F$ is an extension. Define the separable closure $F^{sep}$ of $F$ in $E$ to be the separable elements of $E/F$. Show that $F^{sep}$ is a subfield of $E$. If $E/F$ is finite, show that $E/F^{sep}$ is generated by a tower of $p$th roots. If $E/F$ is algebraic, show that any element of $E$ has some $p^k$th power in $F^{sep}$.

5. Suppose the dihedral group with $2n$ elements acts on the field $k(x)$ with generators mapping $x \mapsto 1/x$ and $x \mapsto \zeta x$ (where $\zeta$ is a primitive $n$th root of unity). Find some $y \in k(x)$ such that $k(y)$ is the fixed field of this group action.

6. Show that the elements $\left\{x_1^{a_1} \cdots x_n^{a_n}\right\}_{0 \leq a_i < i}$ form a basis for $k(x_1, \ldots, x_n)$ over $k(e_1, \ldots, e_n)$ (where as in class $e_i$ is the $i$th symmetric polynomial in $x_1, \ldots, x_n$).