MATH 121 PROBLEM SET 3

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This set is due at noon on Friday February 16 in Jason Lo’s mailbox.

1. Write up a solution to the problem on the midterm on which you scored the least. (If there was a tie, pick the one you think is harder. If it was problem 1, then give full justifications.)

2. Suppose $E/F$ is a finite extension, and $F^{sep}$ is the intermediate field of separable elements of $E$ (over $F$). Show that the degree of $E/F^{sep}$ is a power of $p$. Show that any power of $p$ is possible.

3. Prove that the automorphisms of the rational function field $k(t)$ which fix $k$ are precisely the fractional linear transformations determined by $t \mapsto (at + b)/(ct + d)$ for $a, b, c, d \in k$, $ad - bc \neq 0$ (so $f(t) \in k(t)$ maps to $f((at + b)/(ct + d)$). (Dummit and Foote 14.1 problem 8)

4. Different definitions of normality. Suppose $E/F$ is an algebraic field extension. Show that $E/F$ is the splitting field of a family of polynomials if and only if every irreducible polynomial in $F[x]$ with a root in $E$ splits completely. If $E/F$ is finite, show that these two are equivalent to the statement that any map from $E$ to $F^*$ fixing $F$ must send $E$ to itself.