(This quiz has two pages.) Write your name on the first page of your solutions. Time: 50 minutes. Justify all steps. If you have any questions, please ask. **GOOD LUCK!**

1. (16 points) Suppose \( \alpha : [0,1] \to \mathbb{R}^3 \) given by \( \alpha(t) = (t, t^2, t^3) \), and \( F(x,y,z) = (0, y, x) \). Calculate \( \int \mathbf{F} \cdot d\alpha \).

2. (16 points) In (a) and (b), find if possible a potential function \( \phi(x,y,z) \) defined on all of \( \mathbb{R}^3 \) for each of the following vector fields. If it is not possible, explain why not.

   (a) \( \mathbf{f}(x,y,z) = (2xy - y)\mathbf{i} + (y^2 + 2)\mathbf{j} + (3z)\mathbf{k} \)

   (b) \( \mathbf{g}(x,y,z) = (y^2 + 2)\mathbf{i} + (2xy - y)\mathbf{j} + (3z)\mathbf{k} \)

   (c) Suppose \( \alpha : [0,1] \to \mathbb{R}^3 \) given by \( \alpha(t) = ((e^t - 1)(t^2 - 1), \sin \pi t, 0) \). Calculate \( \int \mathbf{g} \cdot d\alpha \).

3. (16 points) If \( \int \int_R f = \int_1^2 \int_{\sqrt{2-x}}^{x^2} f(x,y) \ dy \ dx \), express \( \int \int_R f \) as an iterated integral where the first (i.e. “inside”) integration is with respect to \( x \). Your answer should be of the form \( \int_? \int_? \ ? \ dx \ dy \).

4. (16 points) Express as an iterated integral the volume of the solid bounded by the surface \( z = 2 - x^2 - y^2 \) and the plane \( z = 0 \). (You don’t need to evaluate the integral.)

5. (16 points) A piece of wire is bent into the circle \( x^2 + y^2 = 4 \) (of radius 2). The density of the wire is \( 10 + x + y \) (in units of mass per unit length). Find the mass of the wire, and the location \( (\bar{x}, \bar{y}) \) of its center of mass.

6. (20 points)

   (a) Define what the statement “\( S \) has content zero” means.

Let \( Q = [0,1] \times [0,1] \). Define \( f(x,y) \) for \( (x,y) \) in \( Q \) by the equations

\[
 f(x,y) = \begin{cases} 
 2 & \text{if } y = x, \\
 1 & \text{if } x = 1/2 \text{ and } y \text{ is irrational,} \\
 0 & \text{otherwise.} 
\end{cases}
\]

*Date: Spring 2001.*
(b) At what points does $f$ fail to be continuous?
(c) Does $\int \int_{Q} f$ exist? Why or why not?
(d) Does $\int_{0}^{1} f(x,y) dy$ exist for all $x$ in $[0,1]$?

**Bonus.** (5 marks) Find the “hypervolume” of the $n$-dimensional version of a triangle (the $n$-simplex) in $\mathbb{R}^n$ bounded by $x_1 = 0, x_2 = 0, \ldots, x_n = 0, x_1 + x_2 + \cdots + x_n = r$. 