18.024 PRACTICE QUIZ I

1. (20 points) Let \( L_1 \) be the line through the point \( P = (a, 0, 0) \) on the \( x \)-axis with direction vector \((-3, 1, -1)\). Let \( L_2 \) be the line \( X = (1, 2, 0) + t(1, -1, 2) \). If \( L_1 \) and \( L_2 \) intersect, find the point \( P \).

2. (24 points) Let \( A \) be a \( k \) by \( n \) matrix; let \( r \) be the rank of \( A \). Answer the following questions in terms of \( n, k, \) and \( r \). (Give answers only.)

(a) What can you say about the dimension of the row space of \( A \)?
(b) What can you say about the dimension of the solution space of the equation \( AX = 0 \)?
(c) What can you say if the system \( AX = C \) fails to have a solution for some \( C \)?
(d) What can you say if you know \( A \) has an inverse?

3. (20 points) Find conditions on \( a, b, c \) that are both necessary and sufficient for the following system to have a solution.

\[
\begin{align*}
x - y + z &= a \\
x + y - 3z &= b \\
\end{align*}
\]

4. (20 points) Find the inverse of the matrix

\[
A = \begin{pmatrix}
2 & 0 & 0 & 1 \\
0 & 1 & -1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}.
\]

5. (16 points) Let \( A \) be a 5 by 5 matrix. Show that if \( A^3 \) has rank less than 5, then \( A \) has rank less than 5.

Another tricky question: Suppose \( A, B, \) and \( C \) are three vectors in \( V_5 \). Can \( 3A + 2B + 4C, A + 4B + 2C, 9A + 4B + 3C, \) and \( A + 2B + 5C \) be linearly independent?