18.014 QUIZ II (PRACTICE)

This quiz has two pages. If you have any questions, please ask.

1. (8 points) Evaluate

$$\lim_{x\to 4}\left(\frac{x^2}{1+\sqrt{x}}\right).$$

Explain what limit theorems you are using.

2. (8 points) Show by example that the conclusion of the extreme-value theorem does not hold if f is only continuous on $[a,b)$ and bounded on $[a,b]$.

3. (10 points) State the mean-value theorem. Make sure you state the hypotheses exactly.

4. (18 points) Let $f(x)$ be a function that is continuous for all x except $x=\pm 2$. Let

$$g(x)=\begin{cases} x^2 & \text{for } x\le 0\\ x^2+3 & \text{for } x>0.\end{cases}$$

(a) For what values of x can you be sure that the function $h(x)=f(g(x))$ is continuous?

(b) Does $g'(0)$ exist? Does $\lim_{x\to 0} g'(x)$ exist? Explain.

5. (32 points) Find $f'(x)$:

(a) if $f(x)=\sin^2(\cos^2 x)$.

(b) if $f(x)=\frac{x^2}{1+\sqrt{x}}$ for $x>0$.

(c) if $f(x)=\int_1^x \frac{dt}{1+t^4}$.

(d) if $f(x)=\int_x^{x^2} \frac{dt}{1+t^4}$.

6. (24 points) One has the following table of values for the continuous functions f and g and their derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	2/3	1	2
1	3	2	0	-7
2	4	5/2	3	5
3	5	4	2	-11

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(a) Find the derivative of $f(g(x))$ at $x = 1$.
(b) Find the derivative of $g(f(x))$ at $x = 1$.
(c) Assume $f'(x) > 0$ for all x, so f has an inverse function h. Find $h'(2)$.

GOOD LUCK!