Math 42 Winter 2006 Practice Final Exam

1. Evaluate each of the following integrals.

   (a) $\displaystyle\int \sin\sqrt{t}\, dt.$

   (b) $\displaystyle\int \frac{1}{x^3\sqrt{x^2-1}}\, dx$

   (c) $\displaystyle\int_0^{\pi} \sin^2 x \cos^2 x\, dx$

   (d) $\displaystyle\int \frac{y^2+1}{y^2-2y+1}\, dy$

2. The speed of a rocket, in m/s, at t seconds after launch, is given by the following table.

t :	0	1	2	3	4	5	6	7	8	9	10
Speed :	0	2	5	9	13	17	20	24	26	28	30

   Estimate the height of the rocket after 10 seconds using the Midpoint Rule.

3. Consider the region R in the xy-plane bounded between $y = 2x$, $y = x^2$, $x = 1$, and $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated by revolving R about the y-axis.

4. Find the arc length of the graph of

$$y = x^{3/2}, \quad 0 \leq x \leq 1.$$

5. A large conical tank has a height of 6 m and a radius at the top of 1 m. The tank is filled to a height of 4 m with oil with a density of 100 kg/m^3. How much work does it take to empty the tank by pumping the oil out the top of the tank? You may use the approximation $g \approx 10$ m/s^2.

6. Find the solution of the initial value problem

$$y' = \frac{e^{-y^2}(t+1)}{yt^2}, \quad y(1) = 2.$$

7. Suppose the growth of a population is modeled by the modified logistic equation

$$\frac{dP}{dt} = \frac{P}{10}\left(1 - \frac{P}{1000}\right)\left(1 - \frac{200}{P}\right).$$

   (a) What are the equilibrium values of the population?

(b) If $P(0) = 400$, use Euler's method with $h = 5$ to estimate the population at time $t = 10$.

8. Determine whether each of the following series converges or diverges.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n^3 + 3n + 2}{n^3 + 6}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n (n+1)}$

9. Find the sums of each of the following series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^n (2n)!}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1 - 2^n}{4^n}$

10. Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n.$$

11. Find a power series expansion (centered at 0) for

$$f(x) = \frac{x}{2+x}$$

and its radius of convergence.

12. Isaac Newton showed that

$$(1 - x^2)^{-1/2} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2} x^{2n}$$

for $-1 < x < 1$.

   (a) Using this formula, find a power series expansion for $\sin^{-1} x$.

   (b) Use your power series from part (a) with $x = 1/2$ to find an infinite series whose sum is π.

13. Use power series expansions to compute

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{\cos x - 1}.$$

14. (a) Find the third-degree Taylor polynomial for

$$f(x) = x^{4/3}$$

about $a = 27$.

(b) Estimate the maximum error involved in estimating f with the Taylor polynomial you found in part (a) for $25 \le x \le 29$.

15. True / False: (You do not need to justify your answer.)

(a) $\displaystyle\int_1^\infty \frac{1}{x^{\sqrt{2}}}\,dx$ is convergent.

(b) If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n b_n\}$ is divergent.

(c) If $\sum c_n 2^n$ is divergent, then $\sum c_n(-3)^n$ is divergent.

(d) The function $f(t) = \dfrac{\ln t}{t}$ is a solution of

$$t^2 y' + ty = 1.$$

(e) If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.