Homework 4

Due: Thursday, May 25

1. Let $C_p = \langle x \mid x^p = 1\rangle$ and $S_n = \langle s_1, s_2, \ldots, s_{n-1}\rangle$. The wreath product $C_p \wr S_n$ is the group given by generators $e_1(x), e_2(x), \ldots, e_n(x), s_1, s_2, \ldots, s_{n-1}$ and relations

$$e_i(x)^p = 1, \quad e_i(x)e_j(x) = e_j(x)e_i(x), \quad e_i(x)s_j = s_j e_{(is_j)}(x),$$

$$s_i^2 = 1, \quad s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}, \quad s_i s_j = s_j s_i, \quad |i-j| > 1.$$

   (a) Determine the conjugacy classes of $C_p \wr S_n$.

   (b) Show that every S_n-module "naturally" gives rise to an $C_p \wr S_n$-module. That is, given an S_n-module, construct a $C_p \wr S_n$-module.

2. Compute the character $\chi_V : S_n \to \mathbb{C}$ for the natural representation of S_n (see homework 3).

3. Let $\rho : G \to GL_n(\mathbb{C})$ be a representation, and define $\Delta_\rho : G \to \mathbb{C}$ by

$$\Delta_\rho(g) = \det(g\rho), \qquad \text{for } g \in G.$$

   (a) Show Δ_ρ is a linear character of G.

   (b) Prove $G/\ker(\Delta_\rho)$ is abelian.

   (c) Suppose $\Delta_\rho(g) = -1$ for some $g \in G$. Show that G has a normal subgroup N such that $|G|/|N| = 2$. Hint: Find a index two subgroup of the image of Δ_ρ.

4. Consider the following table

	1	1	1	1	1	1
	3	-1	0	1	γ	$\bar{\gamma}$
	3	-1	0	1	$\bar{\gamma}$	γ
	5	2	0	0	-1	-1
	7	-1	1	-1	0	0
	8	0	-1	0	1	1

where $\gamma = -\frac{1}{2} + i\frac{\sqrt{7}}{2}$.

   (a) Show that it is not a character table.

   (b) By changing one entry, make it a possible character table.

5. Consider the following character table of a group G.

1	1	1	1	1	1	1	1
1	1	1	1	-1	1	-1	-1
2	-2	-1	1	0	0	$i\sqrt{2}$	$-i\sqrt{2}$
2	2	-1	-1	0	2	0	0
2	-2	-1	1	0	0	$-i\sqrt{2}$	$i\sqrt{2}$
3	3	0	0	1	-1	-1	-1
3	3	0	0	-1	-1	1	1
4	-4	1	-1	0	0	0	0

Find

(a) the order of G,

(b) the sizes of all its normal subgroups,

(c) the size of its center,

(d) the size of its commutator subgroup.