

Homework 2

Due: Thursday, April 26

1. Decompose $\mathbb{C}S_3$ into irreducible modules. Be sure to give explicit bases for the irreducible modules in terms of the natural basis of $\mathbb{C}S_3$. Specify which of the irreducible modules are isomorphic, if any.
2. Give an example to show that Maschke's theorem does not apply to infinite groups. Hint: use and a representation of the group \mathbb{Z} similar to the one we used to show Maschke's theorem does not apply to $\mathbb{Z}/p\mathbb{Z}$.
3. Let G be a finite group.
 - (a) Let $H \triangleleft G$, show that $\mathbb{C}H$ and $\mathbb{C}H \setminus G$ are isomorphic to subalgebras of $\mathbb{C}G$.
 - (b) Show that $\mathbb{C}H \setminus G$ is isomorphic to a submodule of $\mathbb{C}G$, and give an example to show that $\mathbb{C}H$ is not necessarily isomorphic to a submodule of $\mathbb{C}G$.
4. Let V be the irreducible G -module of dimension 2 of D_6 (see question 1). Find (explicit) bases for the vector spaces

$$\text{Hom}_{\mathbb{C}D_6}(\mathbb{C}D_6, V), \quad \text{and} \quad \text{Hom}_{\mathbb{C}D_6}(V, \mathbb{C}D_6).$$

5. Suppose G acts on a (finite) set \mathcal{J} . Let

$$V = \mathbb{C}\text{-span}\{x \mid x \in \mathcal{J}\},$$

or the formal set of linear combinations of elements in \mathcal{J} . Show that the submodule

$$\{v \in V \mid vg = v, g \in G\}$$

has dimension equal to the number of orbits of G on \mathcal{J} .