Math 156: Homework 1

Due: Thursday, April 12

1. Consider the symmetric group $S_4$, the cyclic group $C_8$, and the dihedral group $D_8$. For each group $G$

   (a) Give examples of two nonequivalent and nontrivial representations $\rho$ and $\tau$ (be sure to show they are not equivalent),

   (b) Construct the corresponding $G$-modules $V_\rho$ and $V_\tau$,

   (c) Decide whether the modules are reducible,

   (d) Change bases in the module $V_\rho$ and give the new corresponding representation $\rho' : G \to GL_n(\mathbb{C})$.

2. Show that if $\rho : G \to GL(V)$ is a degree one representation, then $G/\ker(\rho)$ is an abelian group.