

Homework 3

Due: Thursday, May 3

1. How many positive integers less than 1000 have no factor between (and not including) 1 and 10?

2. Prove

$$\sum_{n \geq 0} S(n, n-2)x^n = \frac{x^3(1+2x)}{(1-x)^5}.$$

3. Suppose $\{b_n\}_{n \geq 1}$ is the sequence of Bernoulli numbers (see homework 2). Show that

$$b_n = \sum_{k=1}^n \frac{(-1)^k k! S(n, k)}{k+1}.$$

4. The *girth* of a graph is the length of the smallest cycle in the graph. Let G be a graph such that

(a) the girth of G is at least 5,

(b) $\deg(v) \geq d$ for $v \in V$.

Prove that G has at least $d^2 + 1$ vertices. Can equality hold?

5. Prove that every finite simple graph has at least two vertices with the same degree.