

#3 Compute \sqrt{i} and $\sqrt{-i}$

#1 Show that the system of all matrices of the special form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ combined by matrix addition and matrix multiplication, is isomorphic to the field of complex numbers

#3 Prove that

$$\left| \frac{a-b}{1-ab} \right| = 1$$

if either $|a|=1$ or $|b|=1$. What exception must be made if $|a|=|b|=1$?

#1 Prove that

$$\left| \frac{a-b}{1-ab} \right| < 1$$

if $|a| < 1$ and $|b| < 1$

#1 Express $\cos 3\theta$, $\cos 4\theta$, and $\sin \theta$ in terms of $\cos \theta$ & $\sin \theta$.
#2 Simplify $1 + \cos \theta + \cos 2\theta + \dots + \cos n\theta$ and $\sin \theta + \sin 2\theta + \dots + \sin n\theta$

#5 Prove rigorously that the functions $f(z)$ and $\overline{f(z)}$ are simultaneously analytic

#7 Show that a harmonic function satisfies the formal differential equation

$$\Delta^2 u = 0$$

#4 What is the general form of a rational function which has absolute value 1 on the circle $|z|=1$? In particular, how are the zeros and poles related to each other?
#5 If a rational function is real on $|z|=1$, how are the zeros and poles situated?