Stanford Department of Mathematics Colloquium

October 21
4:15 p.m.
Bldg. 380, Room 380-W.

From branching random walks to Gaussian Free Fields

Ofer Zeitouni, The Weizmann Institute and University of Minnesota

Abstract

Abstract: The (discrete) Gaussian Free Field on a finite graph is the process $\{X_v\}$ with density proportional to $e^{-\sum_{v\sim w}(X_v-X_w)^2}$; it has played an important role in many aspects of contemporary probability theory, as well as in mathematical physics through models for random interfaces and quantum gravity. A natural question relates to the magnitude of fluctuations of the maximum of the field; the planar case, where the graph is a box of side N, is of particular interest.

Branching random walks (BRW's) model the (spatial) evolution of a population, where particles split and then perform independent random motion (on R). As was shown by Bramson in the late 1970's, the behavior of the outliers of the population (ie, the particles that moved farthest from the starting point, after n generations) is determined, in the Gaussian displacement case, by solutions of the Kolmogorov-Petrovsky-Piscounov equation. In the non-Gaussian case, a proof that fluctuations of the maximum are of order 1 was given only very recently.

I will describe surprising links that BRW's have with Gaussian free fields, first passage percolation, and the cover time of graphs by random walks; I will then explain how arguments developed for the study of both Gaussian and non-Gaussian BRW's played a role in a recent resolution of the conjecture that fluctuations of the maximum of the Gaussian Free Field in dimension 2 are bounded.

Based on joint works with E. Bolthausen, J.-D. Deuschel and with M. Bramson.

http://math.stanford.edu/coll/0910/