Stanford Department of Mathematics Colloquium

January 13
4:15 p.m.
Bldg. 380, Room 380-W.

Internal Diffusion Limited Aggregation and the Gaussian Free Field

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Abstract

We study scaling limits of *internal diffusion limited aggregation* ("internal DLA"), a growth model introduced by chemists Meakin and Deutch to study processes like electropolishing, corrosion and etching. In the case of a single point source at the origin, one constructs, inductively, an **occupied set** $A(t) \subset \mathbf{Z}^d$ for each time $t \geq 0$ as follows: begin with $A(0) = \emptyset$ and $A(1) = \{0\}$, and let $A(t+1)$ be the union of $A(t)$ and the first place a random walk from the origin hits $\mathbf{Z}^d \setminus A(t)$.

In the early 1990s Lawler, Bramson and Griffeath showed that the limit shape is a ball. At the same time, Diaconis and Fulton found that the limiting shape from certain multiple sources are described by algebraic curves. In 2009, Levine and Peres gave a systematic treatment showing that the limiting shape is the optimizer for a problem in nonlinear partial differential equations known as the obstacle problem.

In this talk I will describe recent joint work with Lionel Levine and Scott Sheffield in which we prove what we think are the optimal estimates on fluctuations from this deterministic limit. We show that the occupied region with t particles is within distance $O(\log t)$ of the disk of area t in dimension 2 and within $O(\sqrt{\log t})$ of the ball of volume t in higher dimensions. Moreover, the fluctuations satisfy a central limit theorem, formulated in terms of what is known as the Gaussian Free Field, which I will define and explain.