Homework # 7. Extra problems.

1. Let S be a subset of l_2 which consists of sequences $x = \{x_1, x_2, \dots\}$ such that

$$\sum_{k=1}^{\infty} k^2 x_k^2 \leq 1.$$

Show that S is a compact subset of l^2. Hint: look for finite ε-nets in S.

2. Show directly that for any n there exists a polynomial $p_n(t)$ so that

$$||t| - p_n(t)| \leq 1/n$$

for all $t \in [-n, n]$.