Homework # 6. Extra problems.

1. Prove that if $f : \mathbb{R} \to \mathbb{R}$ is continuous and satisfies $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, then $f(x) = xf(1)$ for all $x \in \mathbb{R}$.

2. Give an example of a Riemann integrable function with a countable set of discontinuities.

3. Give an example of a sequence $f_n(x)$ of Riemann integrable functions on $[0, 1]$ that converge point-wise to a limit $f(x)$ which is not Riemann integrable.