

Homework # 2. Extra problems.

1. Consider the alternating series $1 - 1/2 + 1/3 - 1/4 + \dots$ and the corresponding partial sums

$$S_n = \sum_{j=1}^n \frac{(-1)^{j+1}}{j}.$$

(i) Show that the sequence S_n converges. (ii) Rearrange the terms of the series in such a way that any point in the interval $[-1, 1]$ would be a limit point for the new sequence of partial sums.

2. Find a non-negative continuous function $f(x)$ defined for $x \geq 1$ such that

$$\int_1^{\infty} f(x) dx$$

converges but the series

$$\sum_{n=1}^{\infty} f(n)$$

diverges.