Homework # 2. Extra problems.

1. Let K be the set of rational functions, that is, functions of the form $f(x) = p(x)/g(x)$, where $p(x)$ and $g(x)$ are two polynomials. We say that $p(x)/g(x) \geq 0$ if the leading order coefficients of $p(x)$ and $g(x)$ have the same sign. Show that K is an ordered field over the real numbers but is not Archimedean.

2. Find a function $f(x)$ defined on the interval $[0, 1]$ which is not bounded on any interval $(a, b) \subseteq [0, 1]$.

3. Find a function $f(x)$ defined on the interval $[0, 1]$ whose range on any interval $(a, b) \subseteq [0, 1]$ is $[0, 1]$. That is, $0 \leq f(x) \leq 1$ for all $x \in [0, 1]$, and for any $a, b \in (0, 1)$ and any $y \in [0, 1]$ there exists $x \in (a, b)$ so that $f(x) = y$.

4. Let a, b be two positive numbers abd set $x_n = ((a^n + b^n)/2)^{1/n}$. Show that $\lim_{n\to\infty} x_n$ exists and identify this limit.